AN EPITOME OF GEOMETRY,

BEING

A Compendious Collection of the *First*, *Third*, *Fifth*, *Sixth*, *Eleventh* and *Twelfth* Books of *EUCLID*; with their Application to several of the most Useful parts of the *Mathematicks*.

ALSO,

EUCLID's Second Book and *Doctrine* of *Proportion*, Algebraically Demonstrated.

To which is Subjoin'd,

A Treatise of Measuring *Superficies* and *Solids*; *Vulgarly Decimally* and *Practically*, with the *Customs* used by *Artificers* in Measuring their several Works: *Likewise*, Directions for Measuring *Board* and Timber; Making *Vessels* of any Bigness; Taking the Plan of any *Court*, *Yard*, *Garden*, &c. *Also*, The cutting the *Five Regular Bodies*.

By William Alingham,
Teacher of the *MATHEMATICKS*.

LONDON. Printed, 1700. And Sold by the Author in *Channel-Row*, *Westminster*.

TO THE

Right Honourable

John, Earl of *Bridgewater*,
J. Thompſon, Ld. *Haverſham*,
Sir *Robert Rich*, Bar.
Sir *George Rook*, Kt.
Sir *David Mitchel*, Kt.

COMMISSIONERS, for Executing the Office of Lord High Admiral of *England*.

May it pleaſe your Lordſhips,

TO Pardon this my Preſumption in Humbly Preſenting this ſmall piece of *Geometry* to you, being ſenſible that none are more fit to Patronize *Ma*

thematick Sciences than Great Men; and more especially your Lordships, to whose Care so weighty a Concern, as the Management of our *Naval Forces* is committed, upon which (next under God and our Gracious King) the sole Welfare of this Nation depends.

I shall not trouble your Lordships with an Account of the Antiquity and Excellency of *Geometry*, only this I shall say, that no doubt its as old as the World, and that the *Ptolomyes*, who for several Years Sway'd the *Egyptian Scepter*,

Scepter, were great Encouragers of this sort of *Learning*, of which the many Volumes stow'd in the *Alexandrian Library* (had not the Flames destroy'd them) were a sufficient proof; I could also mention divers *Princes* and *Noble Men* of latter Ages, who have been great Lovers of this most Admirable Science.

As to its Utility daily Experience confirms the great Necessity of it, and particularly and principally *Navigation* and *Marine Architecture*, which with out the aid and assistance of this Science would

be wholly imperfect and uſeleſs.

Much more might be ſaid in Commendation hereof, but I ſhall deſiſt, and beg a favourable Cenſure from your Lordſhips of this ſmall yet (I am bold to ſay) uſeful Treatiſe, ſince it has a tendency to promote the Publick Good, which if your Lordſhips ſhall pleaſe to Vouchſafe, will thereby Encourage

Your Lordſhips

Moſt Humble Servant,

WILLIAM ALINGHAM.

TO THE

READER.

THE *great Utility and Excellency of* Geometry, *not only in ſeveral parts of the* Mathematicks, *but alſo in Humane Affairs is manifeſt; For as much as upon it are founded moſt other Sciences, ſome of which are of ſuch Moment to ankind, that without the Knowledge thereof, all our Actions and Tranſactions would be done in Confuſion; So that it needs no* Rhetorick *to recommend it, the great and continual uſefulneſs, with the no leſs pleaſure that attends its Knowledge, are ſufficient Motives, not only to enduce, but Engage one in the Study of it.*

For to this Art all Fabricks, from the most magnificent Structure to the meanest Cottage, owe their Contrivance.

Likewise, *'Tis by help of this Science the Country Gentleman may Survey his Lands, Measure his Timber, Contrive his Buildings, Gardens, Fountains, Aquaducts,* &c. *and if he understood it aright, would not repent* Exchanging *many of his beloved Pastimes, for this greater and more useful Recreation.*

The great Delight and Pleasure also of this Study is not less evident to those who are not sworn Enemies to the Art of thinking, but will allow an Exercise of the mind join'd with that of the Body, to be the more noble sort

of

of Recreation; *For in the consideration of the simplest parts of the Body,* viz. *Points, Lines and Surfaces, variety of delightful Properties are discover'd, which surprize our Mind with unexpected Truth and Conviction, and lay a Foundation to many Noble and useful* Rules *of Practice, not only in several parts of the* Mathematicks, *but also in many Instances of* Common Life.

In short, Time would fail to Enumerate the vast Advantages that attend this most Excellent and almost only Science, to Persons in all Stations, and of all Professions whatsoever, insomuch that no Person can be said to be compleatly accomplish'd, without some Competent Knowledge herein.

But

But no longer to write in praise of Hercules, *or commend that which none but inconsiderate Persons did ever discommend, let me come to speak a little of the following Treatise, which I have presum'd to Entitle* An Epitome of Geometry, *or a Compendious Contraction of the First, Third, Fifth, Sixth, Eleventh and Twelfth Books of* EUCLID; *in the composing of which, I have carefully endeavour'd to rank the Propositions in such order, as to depend upon each other, and from them have drawn their principal* Scholions *and* Corrallaries; *Also in the Demonstration hereof, I have not strictly follow'd one particular Method, but have chang'd it to such that I thought most clear and Natural to conceive.*

To this also is subjoin'd a Treatise of Measuring; *in which I have not only given the Geometrical* Reason *of finding the Content of most of the Bodies herein mentioned, but have also applied them to Practice in the* Measuring *of Artificers Works, and therein have shown the Customs and* Methods *now practised by them.*

In the next place, I have Exhibited *the manner of changing the shape of one Vessel to another, and of what Dimensions in any Form, to make them to Hold any Quantity of Corn, Liquor,* &c.

Lastly, *I have shown how to take the Plan of any Court, Yard, Garden,* &c. *As also the manner of Cutting and Measuring the Five Regular Platonick Bodies.*

These

To the Reader.

These are the Principal Matters in short herein Contained, and what I thought meet to acquaint the Reader withal.

What I have else to say is, that if in what is hereafter delivered, any Mistake be committed, or some things not so clearly expres'd as they might have been; I desire the Courteous Reader to consider, that the Press and our Natures are the Causes of such Effects; Faults ever attending the former, and Frailty the latter; and though undoubtedly it will meet with the Common Fate of all Books, to wit, Cavillers, yet I hope there are others that will kindly accept of what is here offer'd, and freely Pardon the Errors, remembring that they are Humane.

So

To the Reader.

So leaving the following Treatise to the Reader, wishing him in the perusal hereof, as much satisfaction as I had in the writing, and then I am sure his End will be obtained, which that it may, is the hearty desire of their Real Friend,

W. A.

Notes

Notes Explain'd:

=	as a = b	is thus Read.	a *is Equal to* b
+	as a + b		a *more* b
−	as a − b		a *leſs* b
x	as a x b		a *Multiplied by* b
⊐	as a ⊏ b		a *is greater than* b
⊏	as a ⊐ b		a *is leſſer than* b
∠	as ∠ a		*the Angle* a
△	as △ abc		*the Triangle* a b c
√	as √ a		*the ſquare Root of* a
q	as aq		*the ſquare of* a

Theſe four Points (::) placed between four Quantities is the Note of Proportion, as if the four Quantities ſtand in this order, *a* : *b* :: *c* : *d*, they are thus to be read, as *a* to *b* ſo is *c* to *d*.

This Note (⁘) before any parcel of Quantities, ſhews them to be in continual proportion, as ⁘ *a*, *b*, *c*, *d*, &c. are thus to be read. As *a* to *b*, ſo is *c* to *d*.

Two Letters placed in this manner $\frac{a}{b}$ notes Diviſion, and ſhews that *a* muſt be divided by *b*.

Alſo

Also, $a \times b + c$ denotes that a is to be Multiplied by the Summ of b and c. Understand the like by the difference of two Quantities.

Lastly, *W. W. D.* stands for *which was to be demonstrated.*

Take Notice also, 1. That in the Scheems of the following *Theorems*, the Lines drawn for the help of *Demonstration* are generally speck'd.

2. That in these *Theorems* 'tis sufficient to suppose that Parallel Lines can be drawn, Perpendiculars rais'd, Angles fram'd, and other Figures describ'd; the manner of doing them is shewn in the Problems that come after.

3. That for the Convenience of Refferences, I have put all the *Definitions* together; as also all the *Theorems*, numbering them successively: Notwithstanding which, it will be most proper for the Reader to peruse them severally; *that is*, those *Definitions* relating to Plain Geometry, before he enter on the Plain Theorems; and the other Definitions, before those of Solids. And that he may readily know when those Definitions begin, I have incerted these words (*Of Solids*) over the first *Definition* and *Theorem.*

OF

Reader, Notwithstanding the great Care that has been taken in Correcting this Book, yet the Errors following have escap'd Correction.

ERRATA.

PAge 14. Line 18 for *twice* read *thrice*. p. 25. l. 18. for *same* r. *Sum*. p. 18. l. 5. for *its* r. *its*. p. 44. and p. 57. l. 18. for *contract* r. *contact*. p. 177. l. 7. r. *Square Root*. p. 180. l. 4. r. *Measure*. p. 184. l. 20. r. *gives* 270. 17674 *the Circumference*. p. 185. l. 3. r. 49. 63636 *the Diameter*. p. 194. l. 15. r. 42618. l. 18. r. 136. p. 214. l. 18. r. 14721 *Feet*, 756 *Inches*. p. 263. l. 22. r. *Time for Tin*.

ADVERTISEMENT.

AT the House late the Lord *Weymouth*'s, in *Channel-Row Westminster*, are Taught the *Mathematicks* in all their Parts, by the Authour hereof.

OF GEOMETRY.

GEOMETRY, as to its *Name*, ſignifies properly no more than the *Meaſuring* of the *Earth*; but in general, 'tis defined to be the *Science* of *Magnitude*, or *Continued Quantity*, whoſe Parts, though never ſo vaſt, or remote, are by its own *Principles* and *Demonſtrations* underſtood, and exactly meaſur'd; ſo that it hath the whole *Univerſe* for its *Subject*, and is indeed the moſt pure of all the *Mathematick Sciences*.

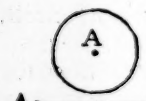
It is principally divided into two Parts, *viz.* *Speculative* and *Practick*. *Speculative* is that which is employ'd about demonſtrating the *properties* of *Magnitudes*: The *Practick* is that which teacheth how to deſcribe and meaſure thoſe *Magnitudes*; and as the firſt hath his *Theorems*, ſo the later hath his *Problems*.

Of this *Science*, I ſhall therefore lay down in the firſt place ſuch *Definitions* and *Principles*, as ought duly to be weigh'd and conſider'd by the *Reader*, before he paſs to the *Demonſtration* and *Practice* of that which follows.

But before we enter on the *Definitions* it may not be amiſs to inform him of the *kinds* of *Magnitudes*, which are *Three*, viz. *Length*, *Breadth* and *Thickneſs*, or a *Line*, a *Superfice*, and a *Solid*. A *Line* being generated by the motion of a *Point*; a *Superfice* by the motion of a *Line*, and a *Solid* by the motion of a *Superfice*; and which way ſoever a *Solid* is moved, it ſtill produceth but a *Solid*. Now, what a *Point*, a *Line* and *Superfice* are, the following *Definitions* will declare unto you.

Definitions.

1. A *Point* is that which hath no part, as the *prick A*.



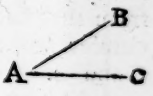
2. A *ſtrait* or *right Line*, is the *ſhorteſt diſtance* betwixt two points, as the line *A B*. Hence two ſides of a Triangle taken together, are longer than the third.

3. A *Superficies* is that which hath *length* and *breadth*, but no *thickneſs*.

4. A *plain Superficies* is the *ſhorteſt Space* that can be deſcribed between one or more *right Lines*, and is bounded by them.

5. A

5. A *plain* or *right lin'd Angle* is made by the meeting of two *right lines* *B A*, *C A*, as the Angle *B A C*. Note, *the middlemoſt of the three Letter points out the Angle*, as by Angle *B A C* is underſtood the Angle *A*.



6. A *Circle* is a *plain Figure* enclos'd by one *line*, call'd a *Periphery*, or *Circumference*, unto which from one Point within the *Figure* all lines drawn are equal, and theſe lines are called *Radius*'s, or *Semidiameters*, as the Figure *A B C D*.

7. And that *point* in the middle of the Figure noted with *E*, is call'd the *Center*.

8. The *Diameter* of a *Circle* is a *ſtrait line* which paſſeth through the *Center*, and is bounded on both ſides by the *Circumference*, dividing the Circle into two equal parts, as the line *A C*.



9. A *Semicircle* is half the whole *Circle*, being contain'd betwixt the *Diameter*, and that part of the *Circumference* cut off by it, as *A B C*.

10. A *Segment* of a *Circle*, is a part cut off by a *right line* (*leſs* than the *Diameter*) drawn within the *Circle*, and the *Segment* may be either greater or leſſer than a *Semicircle*, as the Figure *A* or *Z*.

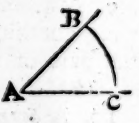


11. A *Sector* is form'd by part of a *Circle*, and two *Semidiameters* drawn from the Center to the Circumference, as the Figures *C* and *B*.



12. Every *Circle*, whether great or small, is suppos'd to be divided into 360 equal parts, call'd *Degrees*, and each *Degree* again into 60 *Minutes*, and every *Minute* into 60 *Seconds*, and so to *Thirds*, *Fourths*, &c. The reason of dividing it into 360 parts, is, because this Number will admit of most equal *Divisions*, as 2. 3. 5. 6. 7. *&c.* And therefore the best way to divide a Circle as aforesaid, is, first to divide it into 4 equal parts, then one of these into 3 equal parts, next one of these into 6 equal parts, and lastly one of these 6 into 5 equal parts, one of which will be a *Degree*, or the $\frac{1}{360}$ part of the whole Circle.

13. The *Measure* of an *Angle*, is an Arch of a Circle described from the *Angular Point*, and lies intercepted betwixt the sides, as the Arch *B C*. Now so many *Degrees* and *Minutes* as are in the Arch *B C*, so much is the measure of it, or of the *Angle A*.



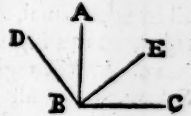
14. A *plain right lin'd Angle* may be *right* or *oblique*.

15. A *right Angle* is that which hath an *Arch* of 90 Degrees for its measure, as the Angle *A B C*.

16. An

16. An *Olique Angle* may be *Acute* or *Obtuſe*.

17. An *acute Angle* is that which is leſs than a *right Angle*, or 90 Degrees, as the Angle *E B C*.



18. An *obtuſe Angle* is that which is greater than a *right Angle*, as the Angle *D B C*.

19. *Right lin'd Figures* are ſuch, as are contain'd under ſtrait Lines.

20. Of *three ſided Figures*, that is, an *Equilateral Triangle*, which hath the three ſides equal to each other, as the figure *A*.



21. An *Iſoſceles Triangle* hath only two of the ſides equal, as the figure *B*.



22. *Triangles* have alſo receiv'd Names from the nature of their Angles; as if the Triangle has one *right Angle*, it is call'd a *right angled Triangle*, as the figure *B A C*, In which *B A* is termed the *Baſe*, *C A* the *Perpendicular*, and the ſide *B C* that ſubtends the *right Angle* the *Hipotenuſe*.



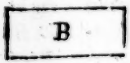
23. If a *Triangle* has no right Angle in it, 'tis termed an *oblique angled Triangle*; and if ſuch Triangle have one *obtuſe Angle*, 'tis call'd an *obtuſe angled Triangle*; But if

no obtuſe or *right Angle*, then an *acute angled Triangle*.

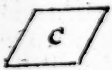
24. Of *Figures* which have four ſides, that is a *Square* which hath all its ſides equal, and Angles right, as the Figure *A*.



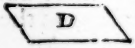
25. A *Rectangle*, or *long Square*, hath all four Angles right and oppoſite ſides equal, as the Figure *B*.



26. A *Rhombus* hath all its ſides equal, and oppoſite Angles equal, but none of them right, as the Figure *C*.

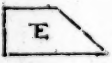


27. A *Rhomboids* hath only its oppoſite ſides and Angles equal, but none of the Angles right, as the Figure *D*.



28. A *Parallelogram* is a four ſided Figure, whoſe oppoſite ſides are *parallel*, as the Figures *A. B. C.* and *D.* in the 24, 25, 26 and 27 *Definitions*.

29. All other *four ſided Figures* are call'd *Trapezias*, as the Figure *E*.



30. Figures having above four ſides, and thoſe unequal, are called *many ſided* (or *Irregular*) *Polygons*, as the Figure *G*.



31. If *Figures* have above four ſides, and thoſe equal, they are called *Regular Poly-*

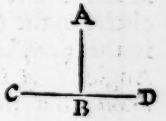


gons,

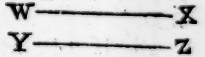
gons, as the Figure *I*, of five equal Sides is term'd a *Pentagon*.

32. If a *Figure* has six equal sides, 'tis named a *Hexagon* or *Polygon* of six equal sides; if seven a *Heptagon* or *Polygon*, of seven equal sides, *&c.*

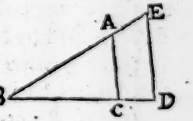
33. If a straight Line *A B* fall upon another straight Line *C D*, and make the Angles *A B C*, *A B D* equal to each other, then those Equal Angles are called *Right Angles*, and the streight Line *A B* is termed a *Perpendicular* to *C D*.



34. *Parallel* or *Equidistant* Right Lines, are such which, if infinitely extended, would never meet; such are the Lines *W X*, *Y Z*.



35. *Simillar* or *Equi-angled* Figures are such which have the Angles of the one severally equal to the Angles of the other, having also the Sides about those Equal Angles proportional; as in the two Triangles *B A C*, *B E D*, the Angle *B* is common, the Angle at *E* in the Greater is equal to the Angle at *A* in the Lesser, and the Angle *D* in the Greater is equal to the Angle *C* in the Lesser; Also *B E : E D :: B A : A C*. So likewise,



wiſe, in all *Quadrangular* or *Multangular* Figures, tho' their Angles are equal, yet the Figures are not ſimillar, unleſs their Correſponding Sides are proportional.

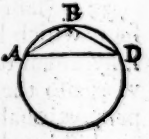
36. *Complements* of Angles or Arches are ſo called in reference to a Quadrant or Semicircle, as the Angle *E A B* is the *Complement* of the Angle *E A D* to a *Quadrant*; but of the Angle *E A C* to a *Semicircle*.



37. The *Diagonal* of any Figure is a Right Line, drawn from the oppoſite Angles thereof; as in the annexed *Trapezium* the *Diagonal Line* is *B D*.



38. If a Point *B* be taken in the Circumference of a Circle, and from it Right Lines *B A*, *B D* be drawn to the Ends of the Right Line *A D*, which cuts off a *Segment*, then the *Angle A B D* contain'd between the Lines *B A*, *B D*, is ſaid to be an *Angle* in a *Segment*.



39. *Segments* are ſaid to be Simillar, when the Angles ſo inſcrib'd are equal.

40. A *Right-lin'd Figure* is ſaid to be *inſcrib'd* in a *Right-lin'd Figure*, when all the Angles of the Figure inſcrib'd touch all the Sides of the other; as the Triangle *D E F* is inſcrib'd

inſcrib'd in the Triangle *A B C*. Alſo the ſaid Triangle is inſcrib'd in the Circle *D E F*, for the Periphery thereof touches all the Angles of the Figure.



41. A *Right-lin'd Figure* is ſaid to be *deſcrib'd* about a *Right-lin'd Figure*, when every Side of the Circumſcribing Figure touches every Angle of the Figure about which it is circumſcrib'd; as the Triangle *A B C* circumſcribes the Triangle *DEF*, alſo the Circle *D E F* circumſcribes the Triangle *D E F*.

42. A Right Line is ſaid to be *apply'd* in a Circle, when the Ends thereof fall upon the Circumference; as the Line *A B* is *apply'd* in the Circle *A B C*. The Line *A B* is alſo called an *Inſcript*; and when ſo apply'd, is ſaid to *cut* the Circle.



43. A Right Line is ſaid to *touch* a Circle, when touching the ſame, and being produced, it cutteth it not; ſo the Right Line *E D*, toucheth the Circle *A B C*.

44. In a Circle *C G D H*, Right Lines *C D*, *G H* are said to be *equally distant* from the Centre *B*, when the Perpendiculars *B A*, *B E*, drawn from the Centre *B*, to them, are equal; and that Line *I K* is said to be fartheſt diſtant from it, on which the longeſt Perpendicular *B F* falleth.



45. The *Height* of any Figure, is a Line drawn from the top of it perpendicular to the Baſe; as *C G* is the height or true breadth of the *Rhomboides BCEF*. It is alſo the height of the Triangle *B C D*.



What Proportion *is.*

46. *Proportion*, *Ratio*, or *Reaſon* is the *Habitude* or *Relation* that one *Magnitude* hath to another according to *Quantity*: As, if 7 be compar'd to 3, then the Reſpect that the *Quantity* of the *Magnitude* 7 has to the *Quantity* of the *Magnitude* 3, is the *Reaſon* or *Habitude* of 7 to 3. In every *Ratio* obſerve, that that *Quantity* which has Reference, or is Referr'd to another, is called the *Antecedent* of the *Ratio*; and the *Quantity* to which it is *referr'd*, is called the *Conſequent*; as in the *Com-*

Compariſon of 7 to 3, 7 is named the *Antecedent*, and 3 the *Conſequent*.

When Quantities *have a Reaſon or Proportion to one another.*

47. *Quantities* are ſaid to have a *Reaſon* or Proportion to one another, when being multiply'd, they exceed another; now there is no *Line* how ſhort ſoever, but if multiply'd, may exceed a longer Line given. But if (any Superficies, as) a Square and *Right Line* be propoſed, the Line cannot be ſo augmented in length as that it can be ſaid to exceed the *Square*, becauſe if infinitely increaſed, it hath ſtill no breadth. Hence therefore 'tis clear, all *Homogenious Magnitudes*, i. e. *Magnitudes* of the *ſame kind*, have a Reaſon or Proportion one to another.

When Quantities *have the ſame Proportion.*

48. *Numbers* or *Quantities* are ſaid to have the ſame *Reaſon* or *Proportion*, when each Conſequent is a like part of its reſpective Antecedent; for no Quantity can be ſaid to be *big* or *little*, but as it is compar'd to another. Suppoſe theſe four Numbers 9. 3. 6. 2. be propoſed, then if 3 is contain'd the ſame

Number

Number of times in 9 as 2 is in 6, I say they are *proportional*; *that is*, they are Quantities that have the same *Proportion*; for there is the same reason of 9 to 3, as of 6 to 2. *Hence note*, that the Quantity of any *Ratio* is known by dividing the *Antecedent* by the *Consequent*, as the Quantity of the *Ratio* of 7 to 3 is expres'd thus $\frac{7}{3}$, or of the *Ratio* of *a* to *b* thus $\frac{a}{b}$; wherefore often, for *Brevity-sake*, the *Quantities* of *Ratio* in the following Theorems are denoted after this manner, $\frac{a}{b} \sqsubset = \text{or} \sqsupset \frac{c}{d}$; that is, the *Ratio* or *Proportion* of *a* to *b* is greater, equal, or less than the *Ratio* of *c* to *d*. From this *Definition* 'tis evident, if *a* contains *b* as oft as it contains *c*, then *b* and *c* are equal; also if *a* contains *b* as often as *c* contains *b*, then *a* and *c* are equal.

Of the Number *of Terms in a Proportion.*

49. *Proportion* consisteth of three Terms at the least, the second whereof doth supply the place of two; and therefore that which I now call *Proportion*, is more rightly term'd *Proportionality*, or the *Similitude* or Likeness of *Proportions*; for as 4 to 6, so is 6 to 9; that is, the Reason of a first Magnitude compar'd to a second, must be like that of a third compar'd to a fourth: So that when there

are

are but three Terms, the *Conſequent* of the firſt *Ratio* is taken for the *Antecedent* of the ſecond; for as 9 : 6 :: 6 : 4; *Proportion* it ſelf generally denoting no more than the *Ratio* betwixt two Quantities.

50. One *Reaſon* is ſaid to be *greater* than another, when one *Antecedent* contains its *Conſequent* more times than the other *Antecedent* doth his *Conſequent*: As if (in the Compariſon of theſe, *A. B. C. D.*) *A* contains *B* more times than *C* doth *D*, then there is a *greater Reaſon* of *A* to *B* than of *C* to *D*.

51. An *Aliquot Part* is a leſſer Number in reſpect of a greater, when it meaſures it exactly: As, 2 is an *Aliquot Part* of 6, becauſe it is contain'd juſt three times in it; and for the ſame Reaſon 3 and 4 are *Aliquot Parts* of 12, for that they are preciſely contain'd in it.

52. An *Aliquant Part* is a leſſer Number in reſpect of a greater, when it doth not meaſure it exactly: As, 3 is an *Aliquant Part* of 7, becauſe it is not contain'd preciſely any Number of times in 7, there being above 2 times 3 in it, and yet not 3 times: Alſo, for the ſame Reaſon, 2 is an *Aliquant Part* of 9.

53. When the *Antecedent* is *Double*, *Treble*, *Quadruple*, *&c.* of the *Conſequent*, 'tis term'd *Double*, *Treble*, *Quadruple*, *&c. Reaſon*; and theſe Names do rather depend upon the *Antecedent*, than the *Ratio* it ſelf.

54. *Num-*

54. *Numbers* are in *Continued* or *Continual Proportion*, when the intermediate Numbers betwixt the first Term and last are taken twice, that is, as *Antecedent* to *Consequent*, so is the *Consequent* taken as *Antecedent* to a fourth Number or Quantity; so 16, 8, 4, 2 are in *Continued Proportion* for as 16 : 8 :: 8 : 4 :: 4 : 2.

55. *Duplicate*, *Triplicate*, *&c. Ratio*, is, when the *Antecedent* is compounded of like *Reasons*: As, Let those four Numbers 324 : 108 : 36 : 12 be in *continual Proportion*, then the *Reason* of the first to the third is *Duplicate* to that of the first to the second: Also the *Reason* of the first to the fourth is *Triplicate* to that of the first to the second. Dr. *Barrow* expresses it thus, $\frac{324}{36} = \frac{324}{108}$ twice, and $\frac{324}{12} = \frac{324}{108}$ twice; which denotes that the Square of the Quote of 324 by 108, is equal to the Quote of 324 by 36; and the Cube of the Quote of 324 by 108, is equal to 324 by 12: So that *Duplicate Ratio* is no more than the Proportion of the first to the third, in three continual Proportionals; and *Triplicate Ratio* the Proportion of the first to the fourth, in four continual Proportionals.

56. *Homologous Sides* are those in any Figure which correspond one to another, in *Numbers* or *Quantities*; the two *Antecedents* are *Homologous Terms*, and so are the two *Consequents*.

57. *Re-*

57. *Reciprocal Figures* are thoſe, whoſe Parts are ſo compar'd, that the *Antecedent* of one *Reaſon* and the Conſequent of the other ſhall be found in the ſame Figure; that is, when the *Proportion* beginneth and endeth in the ſame Figure. In Quantities, 'tis when it beginneth and endeth in the ſame Rank; as if $\left\{\begin{matrix} a:b :: c:d \\ e:b :: c:f \end{matrix}\right\}$ then reciprocally, as $e:a :: d:f$.

Of the Varieties of Proportion.

58. *Alternate Reaſon*, or *Proportion*, is the Comparing of *Antecedent* to *Antecedent*, and *Conſequent* to *Conſequent*; as if $A:B :: C:D$, then alternately compar'd, 'twill be as $A:C :: B:D$.

59. *Inverſe Reaſon* is, when the *Conſequent* is taken as the *Antecedent*, and ſo compared to the *Antecedent*, taken as *Conſequent*; as if $A:B :: C:D$, then inverſly, as $B:A :: D:C$.

60. *Compounded Reaſon* is, when the ſame of the *Antecedent* and *Conſequent* is compared to the *Conſequent* it ſelf; for if $A:B :: C:D$, then by *Compoſition* of *Reaſon*, as $A+B:B :: C+D:D$.

61. *Divided Reaſon* is, when the *Exceſs* wherein the *Antecedent* exceeds the *Conſequent*, is compared to the Conſequent; as if

if $A : B :: C : D$, then by *Division* of *Reason* 'twill be as $A - B : B :: C - D : D$.

62. *Converse Reason*, or *Proportion*, is the *Comparing* the *Antecedent* to the *Excess*, wherein the *Antecedent* exceeds the *Consequent*; as if $A : B :: C : D$, then by *Conversion* as $A : A - B :: C : C - D$.

63. *Mix'd Reason* is the Comparing the Summ of the Antecedent and Consequent to the Difference of the Antecedent and Consequent; for if $A : B :: C : D$, then *mixedly* as $A + B : A - B :: C + D : C - D$.

64. *Proportion* of *Equality* is, when there are taken more *Magnitudes* than two in one Order, and also as many *Magnitudes* in another Order, then by comparing two to two, if there be the same *Reason* of A to B as of E to F, also of B to C as of F to G, and of C to D as of G to H, I say then by Equality of Proportion there will be the same Reason of A to D as of E to H. Or it is a *Comparison* of the *Extream Terms* in both Ranks.

$A.$	$B.$
$C.$	$D.$
$E.$	$F.$
$G.$	$H.$

65. A *Right Line* is said to be *cut* in *Extream* and *Mean Proportion*, whenas the lesser part of the said Line is to the greater, as the greater is to the whole Line; that is, a Rectangle made of the whole Line and lesser Segment, is equal to the *Square* made on the greater Segment.

Of

Of Solids.

66. A *Solid* is that which hath Length Breadth and Thickneſs.

67. The *bounds* of a Solid are Superficies.

68. A *Right Line* is *Perpendicular* to a *Plain*, when it's Perpendicular to all the *Lines* it meeteth in the ſaid *Plain*.

69. One *Plain* is Perpendicular to another, when a Line drawn in one of them is Perpendicular to the other *Plain*.

70. The *Common Section* of two Plains is a Line in both Plains, or it is that *Line* conceived to be drawn on one Plain, by the entrance of another into it.

71. The *Inclination* of one Plain to another, is the *Acute Angle* contain'd by two Lines drawn in each Plain, perpendicular to the *Common Section*.

72. *Parallel Plains* are ſuch, that if infinitely continu'd will never meet.

73. *Like Solids* are terminated under like Plains, equal in Number.

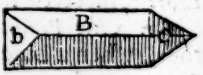
74. *Equal Solids* are terminated under the ſame Number of like and Equal Plains.

75. A *Solid Angle* is made by the meeting of more than two Lines in a Point.

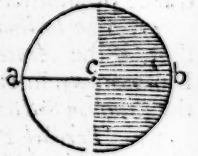
76. A *Pyramid* is a ſolid Figure, whoſe Baſe may be Triangle, Square, Parallelogram, or any other Polygon, but it ſides plain Triangles, whoſe ſeveral Tops meet in a Point, as the Figure *A*.



77. A *Priſm* is a ſolid Figure containing five Plains at the leaſt, the two oppoſite, (*viz. b* and *c.*) of which are *like*, *equal*, *parallel*, and *alike ſcituate*, as the Figure *B*.



78. A *Sphere* is a ſolid Body, produc'd by the moving round a *Semicircle*, upon its *Diameter a b*, ſuppoſed to be fix'd as the Figure *C*, and is therefore bounded by one Surface, to which Lines drawn from the Center or Middle of its Body, are equal.



79. The *Axis* or *Diameter* of the *Sphere*, is a ſtrait Line paſſing through the Center, or that fix'd *Right Line*, about which the *Semicircle* is moved; as *a b*.

80. A *Cone* is a round *Pyramid* in form of a *Sugar-loaf*, generated by the turning round a Right Angled Triangle *a e c*, upon one of the sides *c e*, that includes the Right Angle, as the Figure *D*.



81. The *Axis* of a *Cone*, is that fixt Right Line *c e*, about which the Triangle is moved; The *Base* of the *Cone* the *Circle* described by the moveable Right Line *e a*. And the *Vertex* the *Terminating Point* c.

82. A *Cylender* is a Figure like the *Rolling-Stone* of a Garden, and is formed by the moving round a Right Angled *Parallelogram*, upon one of its sides *a c*, which contains the Right Angle, as the Figure *E*, The *Axis* of which is the said Right Line *a c*, and the *Bases* of the *Cylender* the two Circles described, by the motion of the two opposite sides.



83. *Like Cones* and *Cylenders* are such, whose *Axes* and *Diameters* of their *Bases* are proportional.

84. A *Cube* or *Die*, is a solid Figure contained under 6 equal Squares, as the Figure *F*.



85. A *Tetraedon*, is a solid Figure contain'd under 4 Equal and Equilateral Triangles, as the Figure *G*.



86. An *Octaedron*, is a solid Figure contained under 8 Equal and Equilateral Triangles, as the Figure *H*.



87. A *Dodecaedron*, is a solid Figure contained under 12 Equal Equilateral and Equiangular Pentagons, as the Figure *I*.



88. An *Icoſaedron*, is a solid Figure contained under 20 Equal Equilateral Triangles, as the Figure *K*.



89. A *Parallelepipedon*, is a solid Figure contain'd under six *Quadrilateral* Figures, whereof those which are opposite are parallel, as the Figure *L*. This Figure does also come under the 77th. *Definition*.



90. A *Solid Figure*, is said to be *Inscribed* in a solid Figure, when all the Angles of the Figure Inscribed are comprehended either within the Angles, or in the Plains of the Figure wherein it is Inscrib'd.

91. A

91. A *Solid Figure* is said to be *Circumscrib'd* about a *Solid Figure*, when either the Angles or Sides, or Plains of the Circumscrib'd Figure touch all the Angles of the Figure it contains.

Terms Explain'd.

A *Theo.* or *Theorem* is when something is proposed to be demonstrated.

A *Prob.* or *Problem* is when something is proposed to be done.

A *Prop.* or *Proposition* is used promiscuously, *i. e.* either for a *Theorem* or a *Problem.*

A *Def.* or *Definition* is the unfolding or *Explicating* of the *Nature* and *Affection* of a thing.

A *Post.* or *Postulate* is a grantable request, or such a *Demand* as reasonably cannot be denied.

An *Ax.* or *Axiom* is a *Principle* in any Art, so evident, that it needs nothing but the light of Reason to demonstrate it.

A *Cor.* or *Corollary* is a consequent truth, gained from some preceding Demonstration.

A *Schol.* or *Scholion* is a short Critical Exposition, gained from a former Demonstration, or a *Corollary* wanting an *Explication.*

Hip. or *Hypothesis* is when a thing is supposed, or given so to be, as if it be said *A* is

equal to *B* by *Hypotheſis*; it is as much as to ſay *A* is equal to *B* by Suppoſition.

Con. or *Conſtruction* is the drawing of Lines and framing of Figures, or a preparing the Propoſition for a Demonſtration.

Dem. or *Demonſtration* is the proving a thing by *Definitions* and *Axioms*, and ſo from ſeveral Arguments drawing a Concluſion, that it has that Affection the Propoſition did aſſert.

Lem. or *Lemma* is the Demonſtration of ſome Premiſe, in order to ſhorten a following Demonſtration.

Poſtulates, or grantable Demands.

1. It may not be denied, but that a line from any point to any point may be drawn.

2. That a line may be produced to what length you pleaſe.

3. That from a given Center, a Circle may be deſcrib'd with any diſtance.

Axioms.

1. Things equal to the ſame third, or equal things, are alſo equal one to another.

2. If

2. If to equal things, you add the ſame, or equal things, the wholes ſhall be equal.

3. If from equal things, the ſame or equal things be taken away, the Remainders ſhall be equal.

4. Things *Double*, *Treble*, *Quadruple*, &c. or a *Half*, a *Third*, a *Quarter*, *&c.* of the ſame or equal Things, are alſo equal one to another.

5. The whole is greater than any of its parts, but equal to all of them taken together.

6. Thoſe things which fit one another, and agree in all their parts, are equal.

7. If one whole be *double*, *treble*, *&c.* to another, and that which is added to, or taken away from the firſt *double*, *treble*, *&c.* to that which is added to, or taken away from the Second, the Summ or Remainder of the Firſt, ſhall be *double*, *treble*, *&c.* to the Summ or Remainder of the Second.

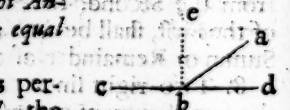
8. Two right lines cannot have one and the ſame Segment common to them both.

9. Two right Lines include no ſpace.

THEOREMS IN *Plain GEOMETRY.*

THEO. I.

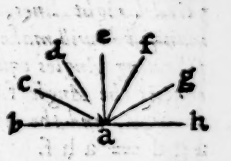
IF *a ſtrait Line* a b *fall on another ſtrait Line* c d, *it makes either two right Angles, or two Angles equal to two right.*



Dem. If *a b* is perpendicular to *c d*, the ∠ *a b d* = *a b c* (*def.* 33.) and therefore both right, If not, let *e b* be a perpendicular, then is the Angle *a b d* + *a b e* equal to one right (*def.* 33. *ax.* 5.) to which add the Angle *e b c*, and they all make two right, *Which Was to be Demonſtrated.*

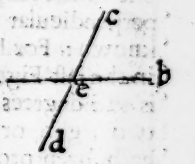
Cor. 1.

Cor. 1. Hence, if never ſo many right lines, as *c a*, *d a*, *e a*, *f a*, *g a*, meet together at one point, and on the ſame ſide of any ſtrait line as *b h*, all the Angles form'd by them at that point, are only equal to two right (*ax.* 5.)



2. If to any *ſtrait line*, as *e a* and point therein *a*, two *right lines b a*, *a h* be drawn on contrary ſides, making the Angles *b a e*, *e a h* equal to two right, then ſhall thoſe lines make one ſtraight line.

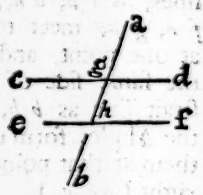
Schol. 1. *From this* Theo. *'tis evident, that if two right lines cut through each other, the oppoſite Angles at the point of cutting are equal*: For the ∠ *a e c* + *a e d* is equal to two right; by the *preceding Theo.* ſo alſo is the ∠ *a e d* + *d e b*; therefore (*ax.* 3.) ∠ *a e c* = *d e b W. W. D.*



Cor. Hence all the Angles made about one point, are equal to four right.

Schol. 2.

Schol. 2. *If a right line cut through two parallel right lines, 'tis manifeſt 'twill make the following Angles equal; that is the Angle* f h g = c g h, *and the Angle* a g d = a h f. For two parallel lines are but the bounds or outſides of one broad line and therefore from the foregoing Demonſtrations, 'tis evident that the ∠ *f h g* = to *c g h*, and the ∠ *a g d* = *a h f W. W. D.*



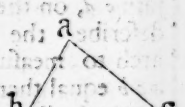
Uſe. 'This *Theo.* is of great uſe, both in 'Plain and Spherical *Trigonometry*, for when 'one of the Angles made by the falling of a 'perpendicular is known, the other is alſo 'known: For Example, if the Angle *a g d* 'in the laſt Figure be 60 degrees, the *a g c* 'is 120 degrees, the complement thereof to 'two right, or 180 degrees, which is (as 'hath been proved) the quantity of both 'theſe Angles, and is therefore found by 'ſubſtracting 60 degrees therefrom.

'Alſo from the laſt *Cor.* is determined 'what Polygons may be joined together for 'Paving, and by it we find that either 6 'Triangles, 4 Squares or 3 Hexagons will 'perform the buſineſs; for the 6 Angles of 'an Equilateral, the 4 of a Square, and 3 of 'a Hexagon are equal to 4 right.

THEO.

THEO. II.

If in two Triangles a b c, d e f *there be two sides* a b, a c *of the one, equal to two sides* d e, d f *in the other respectively, and the included* ∠ a *of the one, equal to the included* ∠ d *of the other, then shall the rest of their parts, and consequently the two Triangles be equal one to the other.*

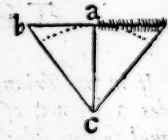


Dem. For lay the point *d* on the point *a*, and the right line *d e* on the right line *a b*, then shall the point *e* fall on *b*, for *d e* = *a b* (*Hyp.*) and the right line *d f* upon the right line *a c*, because the Angle *b a c* = *e d f* (*Hyp.*) also the point *f* will fall on the point *c*, for *d f* = *a c* (*Hyp.*) and therefore (*ax.* 6.) will the rest of the parts be equal, *that is* ∠ *a b c* = *d e f* and ∠ *a c b* = *e f d*, also *b c* = *e f*, and consequently both Triangles, *W. W. D.*

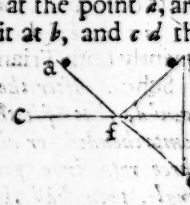
Schol. *After the same manner it might be prov'd, that if either two Angles and a side comprehended, or all three sides, be equal to the three respective parts or sides of another Triangle, then shall the two Triangles be equal to each other.*

Use,

Uſe. 'Hence may an inacceſſible diſtance
'(ſuppoſe *a d*) be meaſur'd, for from *a* erect
'*a c* as long as you pleaſe,
'perpendicular to *a d*,
'join *c d*, on the point *c*
'deſcribe the prickt
'arch to meaſure the ∠ *a c d*, and make
'*a c b* equal thereto, then draw *c b*, which
'meets the line *d a* produc'd in *b*; now be-
'cauſe in the two Triangles *b a c*, *d a c*,
'there is a common ſide *a c*, and two An-
'gles including this ſide in one, *viz.* *b a c*,
'*b c a* equal to the two Angles *c a d*, *a c d*
'including the ſaid ſide in the other, there-
'fore ſhall the reſt of the parts be equal, and
'conſequently *b a* = *a d*, ſo that meaſuring
'the acceſſible line *b a*, you have the length
'of the inacceſſible line *a d*.



'Alſo by this *Theo* we are taught how to
'ſtrike a Bowl at *Billiards*, ſo that by its re-
'flection, it will hit any other Bowl propo-
'ſed: Suppoſe one Bowl at the point *a*, and
'the other you would hit at *b*, and *c d* the
'the *Billiard Table*, ima-
'gine to the point *b*, the
'line *b e* be drawn per-
'pendicular to *c d*, till
'*d e* = *d b*, and join *a e*,
'then I ſay that *f* is the
'point, to which if the Bowl at *a* be direct-
'ed, it will by reflection carry it to *b*. For



in

'in the Triangles *b f d*, *e f d* there are two 'sides and an Angle reſpectively equal, but '(by *Schol.* 1. *Theo.* 1.) the ∠ *a f c*=*e f d*, 'but the ∠ *e f d*=*d f b*, and therefore the '∠ *d f b*=*a f c*, and therefore the point 'of the Table is *f*, which if ſtruck from *a* 'will reflect the Ball to *b*.

'Likewiſe from hence it is eaſie to con-'ceive, how Light may be transferr'd from 'one place to another, *viz.* by only placing 'ſeveral Glaſſes, ſo that the Angle of Inci-'dence may be equal to the Angle of Re-'flection, by which means the light of a 'Candle may be brought out of one Room 'into another.

THEO. III.

In any plain Triangle as a b c, *equal Angles as* a *and* c *are ſubſtended by equal ſides.*

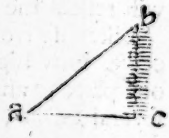
Dem. If you deny it, let one ſide be longer than the other, ſuppoſe *c b* ⊏ *a b*, make *c d* = *a b* by drawing the line *a d*. Now becauſe in the Triangles *b a c*, *d a c* there is a common ſide *a c*, alſo *c d*=*b a* and the contained Angle *d c a*=*b a c*, therefore (*Theo.* 2.) is the



the Triangle *a d c* = *a b c* a part to the whole, which is impoſſible.

Cor. Hence if in any Triangle the three Angles are equal, then are the three ſides alſo equal, *and contra.*

Uſe. 'By this *Theo.* the height of any Ob-'ject from its ſhadow may 'be found; for watch till 'the Sun hath 45 *Degrees* 'of height, and at that In-'ſtant meaſure the length 'of the ſhadow of any Ob-'ject, and the length will be equal to the 'height of the Object, for the Angles *a* and *b* 'made by the Sun's Ray *a b*, with the Ob-'ject *b c*, and *ground line a c* are equal, and 'therefore (by this *Theo.*) the ſides *a c*, *a b* 'are equal that ſubtend them.



THEO. IV.

The three Angles a, b, c *of any plain Triangle, as* a b c *are equal to two Right.*

Dem. Produce any ſide as *b c*, and draw *c e* parallel to *a b*, then is the Angle *a c e* = *b a c* (*Schol.* 2. *Theo.* 1.) and by the ſame the ∠



e c d

$e c d = a b c$, now if to the whole Angle $a c d$ you add the Angle $a c b$ it is evident (*Theo.* 1. *ax.* 1.) that the three Angles, viz. $a + b + a c b$ are equal to two Right. *W. W. D.*

Cor. 1. Hence it follows, that if one ſide of a Triangle is produced, the outward Angle formed thereby, is greater than either of the inward oppoſite Angles of the Triangle.

2. That if one Angle in a Triangle is right, the other two taken together are equal to a Right, but ſeverally acute.

3. That if one Triangle hath two Angles ſeverally or together, equal to two Angles in another Triangle (ſeverally or together) then ſhall the remaining Angle of the one, be equal to the remaining Angle of the other; ſo alſo if two Triangles have one Angle of the one, equal to one Angle of the other, then is the Summ of the remaining Angles in the one, equal to the Summ of the remaining Angles in the other.

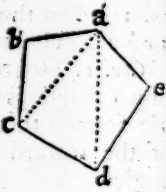
4. All the Angles of an Equilateral Triangle, and the two Angles at the baſe of an *Iſoſceles* Triangle are acute.

5. That one Angle of an Equilateral Triangle is $\frac{2}{3}$ of one right Angle; for $\frac{1}{3}$ of two right Angles, is $\frac{2}{3}$ of one right.

Schol. 1. *By the help of this* Theo. *you may know how many Degrees there are in any right lin'd Figure.*

Sup-

Suppose in the Figure *a b c d e*, first by Diagonals cut the Figure into Triangles, which is three, for the number of Triangles is always lesser by two, than the number of sides in the Figure. Now the Angles of the Triangles make all the Angles of the Figure, which



because there is three, make Six right Angles by the preceding *Theo.* which 6 being multiplied by 90, the degrees in one right Angle will give you 540, the Degrees in the whole Figure.

Cor. Hence all right lin'd Figures of the same number of sides, have the Summs of their Angles equal.

Schol. 2. *Hence may the Degrees of the Angle of any Regular Polygon be known:* For dividing the number of Degrees in all the Angles of a *Polygon*, by the number of the Angles, the Quote will give you the Degrees contained in one Angle; thus proceeding, you will find the Angle of an Equilateral Triangle 60 Degrees, the Angle of a Square 90, the Angle of a Pentagon 108 Degree.

Use. 'Many and great are the uses of this 'Theo. for 'tis by this Astronomers determine 'the *Parallax, which is an Angle subtended by* 'the *Semidiameter of the Earth.* As let *a* de-

note

'note the Earths Center, and from *b*, a point 'in its Surface, let be taken by 'Observation the Angle *d b c*, 'which is the distance of the 'Star from the *Zenith*. Now 'tis plain, that if the Earth 'were transparent, and the 'the Star observed from *a* its 'Center, the Angle *c a d* would 'be made, which is less than 'the Angle *c b d*, by the Angle *a c b*, for 'c b d = c a d + a c b, *by the preceding Theo.* 'so that the Angle *a c b* will be equal to the 'excess of the Angle *c b d* above the Angle 'c a d. If therefore I can by any Artifice 'find how far distant from the *Zenith* the 'Star ought to appear at the Center of the 'Earth, at the same time 'tis observ'd from 'the Surface, I may then find the Quantity 'of the Angle *a c b* which is the Angle of 'Parallax, and is subtended by *b a* the Semi-'diameter of the Earth.



'Again, by the Second. *Schol.* Surveyors 'have a method to prove whether (when in 'Surveying a Field by going round it) they 'have taken the Angles right, for having 'observed them by an Instrument, find their 'Sum by adding them together; for Ex-'ample, let the Figure *a b c d e* (*Schol.* 1.) 'be a Field, whose Angles had been taken: 'divide the Figure into Triangles, and find by 'the said *Schol.* the number of Right Angles

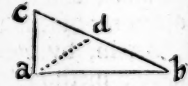
'contained in the Figure, then if the Sum
'of those so found, be equal to the Sum
'of those which were taken, the Work i
'right, otherwise not.

'Lastly, By it may be Constructed an
'Regular Polygon upon a given Line, whic
'is of great use in Fortification; for in th
'*Ichnographick* Projection of any *Regula*
'*Fort*, the first thing to be done is to de
'scribe a Regular Polygon upon a give
'side.

THEO. V.

The greatest side c b *of any Triangle* c a b *subtends the greatest Angle* a.

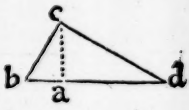
Dem. Take from *c b*, *c d* = *c a* and join *a d*. The Angle *c d a* = *c a d* (*Theo.* 3.) But (*Theo.* 4. *Cor.* 1.) the Angle *c d a* ⊏ *c b a*, that is the Angle *c a d* ⊏ *c b a*; and therefore the Angle *c a b* ⊏ *c b a* (*ax.* 5.) In like manner it may be proved that the Angle *c a b* is greater than the Angle *c*. *W. W. D.*



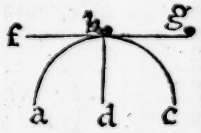
Cor. Hence the greatest Angle will always be subtended by the greatest side.

Use.

Uſe. 'The principal uſe of this *Theo.* is to 'prove that from one and the ſame point, 'and to the ſame *right line*, there can be but 'one perpendicular drawn, and alſo that 'this perpendicular is the ſhorteſt line or 'neareſt diſtance from that point to the *right* 'line; to which it is a perpendicular; For 'Example, let *c a* be perpendicular to *b d*, I ſay then, that *c a* will be 'leſs than *c b*, becauſe it 'ſubtends a leſſer Angle; 'For the Angle *b a c* be-'ing right, the Angle *b* is acute, (*Theo.* 4. *Cor.* 2.) and therefore *c a* ⊐ *c b*. After the 'ſame manner it might be proved, that *c a* 'will be leſs than any other line drawn 'from *c* to any point in the line *b d*, ex-'cept *a*.



'Alſo from hence it may be proved that 'a Bowl exactly round cannot reſt on any 'plain parallel to the Horizon, but upon 'one certain point; for let *a b c* repreſent the ſurface of the Earth, and *d* its center, then let any plain ſuppoſe *f g* be a tangent to the Earths ſurface. Now becauſe 'tis the nature of all heavy Bodies not to reſt when they may deſcend, and the Bowl at *g* having no-

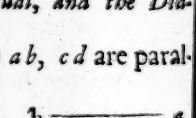


'thing to hinder its nigher approach to the 'Center, must (by consequence) continu-'ally descend till it comes to the point *b*, 'where it will rest, because it is then nigher 'the Center *d*. Hence it is also evident, 'that a liquid Body must flow from *g* to *b*; 'and that the surface of all *Stagnant* Liquors 'is Spherical.

THEO. VI.

The opposite sides ab, dc *and* bd, ac *in a* pgrm. *are equal, and the oppsite Angles* a, d, *and* abd, acd *are also equal, and the Diameter* bc *bisects it.*

Dem. Because (*def.* 28.) *ab*, *cd* are parallel, therefore (*Theo.* 1. *Schol.* 2.) the ∠ *abc* = *dcb*, also for the same reason the Angle *bca* = *dbc*, and the side *bc* common, therefore (*Schol. Theo.* 2.) the Triangle *abc* will be equal to *bcd*, and consequently the side *ab* = *dc* and *ac* = *bd*. *W. W. D.*



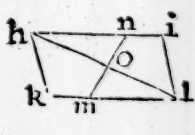
Schol. *Hence it might easily be proved that if two equal and parallel lines,* (suppose *ba*, *dc* in the last Figure) *be joined together with*

two

two other right lines (ac, bd) *then are thoſe Lines alſo equal and parallel.*

Uſe. 'Upon this *Theo.* is grounded the 'method of drawing all manner of parallel 'lines, as in the foregoing Figure: Let it be 'required to draw through the point *b*, a 'parallel to *dc*, join *bd*, from *b* draw *ba* '= *dc*, and from *a* let be drawn *ac* = *bd* 'and the thing is done, for becauſe *bd*, and '*ac* are equal, and *ba* = *dc*, therefore (*pre-*'*ceding Theo.*) *bcda* is a *pgrm.* and therefore 'hath its oppoſite ſides *ba*, *dc* parallel.

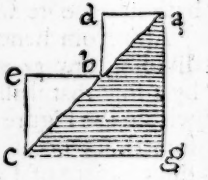
'Alſo from hence a Method is learnt of 'dividing any *pgrm.* into two equal parts, 'by a line that ſhall paſs through any point 'within the Figure (which Surveyors have 'ſometimes occaſion to perform, in divi-'ding a piece of Land into two equal parts, 'yet ſo as the Fence may paſs through a '*Pond* or *Lake* which is in ſome part there-'of.) As let it be required to divide the '*pgrm. hikl* into two equal parts by a 'line that ſhall paſs through the point *m*, di-'vide the Diameter e-'qually in *o*, and draw 'the right line *mon*, 'this Line ſhall di-'vide the ſaid Figure 'into two equal parts.



'For the Triangle *hno* = *lmo*, there being 'in either, two Angles and a ſide compre-'hended by them ſeverally equal, alſo the

'Triangle *h k l* = *h i l*, therefore ſhall the 'Trapezium *h m n k* be equal to the *Trapezium n i l m* (*ax.* 2 and 3.) ſo that the *pgrm. h i k l* is biſected by the line *m n*, which 'was to be done.

'Laſtly. By the *Schol.* of this *Theo.* may be 'meaſured the Horizontal Lines and Perpendicular Heights of *Mountains*, which 'lines are hidden by their *ſolidity*. For Example, let the Mountain *a g c* be to be 'meaſur'd; take a very large ſquare as *a b d*, 'and placing one end thereof at *a* in ſuch 'ſort, that the other 'ſide *d b* may be perpendicular to the Horizon, then meaſure 'the ſides *a d*, *d b*; do



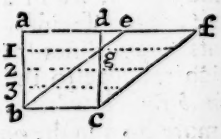
'the like again at the point *b*, and meaſure 'the lines *b e*, *e c*, then if you add the ſides '(*d a*, *e b*) which are parallel to the Horizon together, the Summ will give the 'length of the horizontal Line *c g*, and the 'ſides (*b d*, *e c*) perpendicular to the Horizon being added, gives the perpendicular 'height *a g*. Note, if the ſide of the ſquare '*d a* be made to ſlide higher or lower on the 'ſide *d b*, the thing is practicable.

THEO.

THEO. VII.

Parallelograms b d, b f *ſtanding on the ſame (or an equal) baſe* b c, *and betwixt the ſame or equidiſtant Parallels* a f, b c, *are equal.*

Dem. For *a b* = *d c* (*Theo.* 6) and *a d* = *b c* = *e f* by the ſame, add *d e* common, then is *a e* = *d f* (*ax.* 2.) Alſo the Angle *e a b* = *f d c*, therefore the Triangle *f d c* = *e a b*, take away the common Triangle *d g e*, there remains the *Trapezium c g e f* = *a d b g* (*ax.* 3.) add the Common Triangle *b g c*, then is the *pgrm. b d* = *b f* (*ax.* 2.) *W. W. D.*



Cavalerius *his Demonſtration of the former Propoſition by the Method of* Indiviſibles.

LET the former *pgrms. b d*, *b f* be propoſed having the ſame Baſe *b c*, and being betwixt the ſame parallels *c f*, *b e*; divide the ſide *a b* in as many parts as you pleaſe, ſuppoſe into 4, and from thoſe points draw lines parallel to the Baſe, which produce to the out ſide *c f* of the *pgrm. b f*. Now 'tis plain that there will be no more lines in the one than there is in the other, neither will they differ in length, being all equal to the Baſe *b a*, nor will they be cloſer in the one than in the other, for the lines are all parallel to the Baſe, and conſequently one to another, therefore will the two *pgrms.* be equal, becauſe compounded and made up of like and equal parts.

Uſe, 'This *Theo.* preſents us with the 'method of meaſuring any *pgrm.* (whether 'Square, Rectangle, Rhombus, or Rhom-'boids, for they all come under that Deno-'mination (by their *Def.* and *Theo. 6.*) as 'thus. Imagine in a Rectangled *pgrm.* the

'the ſide *a b* to be carried perpendicularly 'through the whole 'Line *b c*, or elſe *b c* 'through *a b*, then 'ſhall this motion pro- 'duce the Area of the '*pgrm.* for it takes 'up the whole ſpace '*a d c b*, paſſing through every phiſical point 'therein.



'Hence a Rectangle is ſaid to be produ- 'ced by the multiplication of two Lines, 'as if *a b* be 5 and *b c* 3, draw 3 into 5: 'and 'twill produce 15 for the ſaid rect- 'angle.

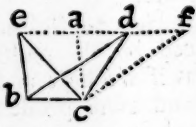
'This being ſuppoſed, the Dimenſion of 'any oblique *pgrm.* as *e f* may be found, by 'multiplying the Baſe *b c* into the true 'height (which is a Line let fall perpen- 'dicular to the Baſe from ſome point *e*, 'in the oppoſite ſide) as *e n*, the product 'of which giveth the Area of the *pgrm. e f*; 'for by the preceding *Theo.* 'tis equal to a 'right *pgrm.* having an equal baſe and 'height.

'Thus if *e n* were 6 and *b c* 5 the Area 'of the *Rhomboids* or *pgrm.* will be 30. 'After the ſame manner is the *Rhombus* 'meaſured, by dropping a perpendicular 'from an Angle to the ſide oppoſite, and 'multiplying any one ſide by the ſaid per- 'pendicular. The *Square* is meaſured by

'mul-

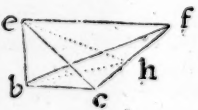
'multiplying one of its sides into it self, as
'if the side of a Square be 4, the Area of it
'will be 16.

THEO. VIII.



Triangles b c e, b c d *standing on the same (or an equal) base* b c, *and betwixt the same or equidistant parallels* e f, b c *are equal, and contrary* i. e. *equal Triangles* b c e, b c d *on the same base (and side)* b c, *are also between the same parallels* e b c.

Dem. Compleat the *pgrms. b a, b f* by drawing *c a, c f* parallel to *b e, b d.* Then (*Theo.* 6.) *e c* bisects the *pgrm. b a,* as also *d c* the *pgrm. b f,* therefore the Triangle *b c e* $= \frac{1}{2}$ *pgrm. b a* $= \frac{1}{2}$ *pgrm. b f* (*Theo.* 7. *ax.* 4.) $=$ $\triangle$ *b c d. W. W. D.*



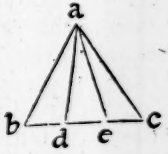
Secondly, If *e f* is not parallel to *b c*, draw *e h* parallel to it, and joyn *h b*, then is the $\triangle$ *b h c* $=$ $\triangle$ *b e c* (by this *Theo.*) $=$ *b f e* (*Hyp.*) *Which* (*ax.* 5.) *is impossible.*

The

The like consequence would follow, if *e h* was drawn above *e f*.

Cor. Hence 'tis evident, that a *pgrm.* betwixt the same parallels, and on the same (or an equal) base with a Triangle, is double to it.

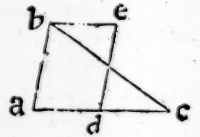
Use. 'By this *Theo.* a practical way is 'found to divide a triangular Field into any 'number of equal parts; 'For Example, divide the 'Triangle *a b c* into three 'equal parts, divide any 'side into three equal parts, 'as *a b c* in the points *d*, *e*, 'and from the opposite ∠ *a* 'draw *a d*, *a e*, then are the Triangles *b a d*, 'd a e, e a c equal (preced. *Theo.*) for they 'have the same height, and stand on 'equal Bases.



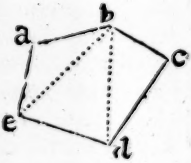
2. 'This *Theo.* doth also present us with 'the reason of the Rule for measuring a 'Triangle, which Rule is to multiply half 'the base by the whole perpendicular, for 'since (*Theo.* 7.) the whole base by the 'whole perpendicular produces the Area 'of a *pgrm.* which (*Cor.* of this *Theo.*) is 'double to a Triangle of the same (or an 'equal) base and height, therefore the 'whole of one Line by the half of the other, 'will produce but half the Area of the 'pgrm. which must therefore by the said 'Cor. be the Area of the Triangle.

3. From

3. 'From this Propoſition may eaſily be 'ſeen how a *pgrm.* may be made equal to 'any Triangle, as let it be required to make 'a *pgrm.* equal to the 'Triangle *a b c*, having 'biſected the baſe in *d*, 'from *d* draw a paral- 'lel to *a b*, making *d e* '= *a b* and join *b e*, 'then is the *pgrm. a e* 'equal to the Triangle *a b c*, as is manifeſt 'from the preceding *Theo.*



4. 'It is alſo by this Propoſition, that 'the meaſuration both of the *Circle*, and 'alſo of all right lin'd Figures is performed. 'For Example, Let the 'Figure *a b c d e* be pro- 'poſed to be meaſured, 'draw Lines from An- 'gle to Angle, and ſo 'reduce the Figure into 'Triangles, then mea- 'ſure each Triangle ſe- 'verally (by leting fall a perpendicular, and 'and meaſuring both it and the baſe by a 'Scale of equal parts) and collect all thoſe 'Area's into one Sum, ſo ſhall this total be 'the Area or Content of the whole Figure.



5. 'If it were required to meaſure a 'Circle, Imagine the Circumference divided 'into 100000 (or more) equal parts, now 'I ſay the difference betwixt one of thoſe 'Arches

'Arches and its ſubtenſe, will be ſo ſmall, 'that ſcarce any operation will require 'greater exactneſs, but that the one may 'be taken as equal to the other, the diffe-'rence being inſenſible: If therefore Lines 'are conceived to be drawn from the Cen-'ter, there will be 100000 Triangles con-'ſtituted, the Summ of the Area's of which 'Triangles will be equal to the Area of the 'Circle, and the Summ of all the Baſes e-'qual to its Circumference, and therefore 'by the preceding *Theo.* if the half Summ 'of all the Baſes (*i. e.* half the Circumfe-'rence) be multiplied by the whole per-'pendicular, which without ſenſible Error 'may be taken for the Semi-diameter of the 'Circle, the Product will be the Area of all 'the Triangles, and conſequently the Area 'of the whole Circle.



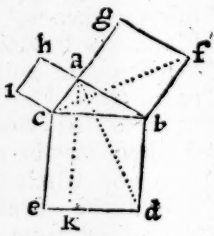
'Hence 'tis evident how a 'Triangle may be made equal 'to a given Circle. For having 'drawn the Semi-diameter '*b a*, erect from *a* the per-'pendicular *a c*, and make it 'equal to the Circumference 'of the Circle, then join the 'points *c*, *b*, by the right line '*c b*, ſo ſhall this Triangle '*a b c* be equal to the Circle '*a b d*, as may plainly appear from what be-'fore has been delivered.

THEO.

THEO. IX.

In all Right Angled Triangles as b a c, *the Square of the Hypotenuſe* b c *is equal to both the Squares* h c, g b *of the other two ſides added together.*

Dem. Let the Figures *b e*, *g b*, *h c*, be the Squares of the three ſides *b c*, *b a*, *a c*, and let *a k* be drawn parallel to *c e*, as alſo the lines *f c*, *a d*. Then becauſe the Angles *c b d*, *f b a* are equal (being both right) add to each the Interjacent Angle *a b c*, then is the $\angle f b c = a b d$ (*ax.* 2.) but $f b = a b$ (*def.* 24.) and $b c = b d$, therefore the Triangles *f b c*, *a b d* are equal (*Theo.* 2.) but the *pgrm. b k* is double to the Triangle *b a d* (*Cor. Theo.* 8.) alſo the Square *b g*, is double to the Triangle *f b c*, for *g c* is a ſtraight Line, therefore is the *pgrm. b k* equal to the ſquare *b g* (*ax.* 4.) After the ſame manner it might be proved that the *pgrm. k c* is equal to the *Square c b*, therefore (*ax.* 2.) the whole

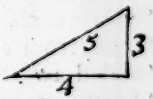


ſquare

ſquare *b e* of the ſide *b c* ſubtending the right Angle, is equal to both the Squares, *viz.* *b g*, *c h* made on the ſides *b a*, *a c* containing the right Angle *W. W. D.*

Uſe. 'This *Theo.* is of ſuch great uſe, not 'only in ſeveral parts of the *Mathemaaticks*, 'but alſo in many Mechanick Operations, 'that 'tis ſaid *Pythagoras*, the firſt finder there-'of, in gratitude to the *Muſes* ſacrific'd a 'Hundred *Oxen* to them, he ſuppoſed it (as 'by this it ſeems) beyond the power of 'bare humane Invention; neither was this 'his eſteem thereof ſo Irrational, as to ſome 'perhaps it may appear, ſince 'tis the foun-'dation of many conſiderable *Practices* in 'the Mathematicks.

'As Firſt, *Trigonometry* which cannot be 'performed without the aid and aſſiſtance 'hereof, for if in any right angled Triangle, 'the Hypotenuſe and a leg be given the o-'ther by this *Theo.* may be found, for ha-'ving ſquared the *Hyp.* which for Example, 'I ſhall ſuppoſe 5, and likewiſe the given 'Leg, which let be 3, then taking 9 the 'Square thereof from that of 5 which is 25, 'the Remainder will be '16, the ſquare Root of 'which is 4, the length 'of the other ſide; Thus 'likewiſe if the two legs



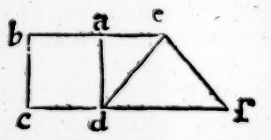
'were given, the length of the *Hypotenuſe* 'might be found, by ſquaring each Leg,

'and

'and adding the two Products together, to 'wit, 9 and 16 the Summ of which is 25, 'and extracting the ſquare Root of this 'Summ, it will give 5 the length of the 'Hypotenuſe required.

'*Secondly*, 'Tis by help of this *Theo.* that 'the Tables of natural *Signs* and *Tangents* are 'Calculated, by which, both plain and 'ſpherical Triangles have their Solution.

'*Thirdly*, By this *Theo.* we not only can 'make a Square, double, treble, quadru-'ple, *&c.* to any one given, but alſo con-'ſtruct a ſquare equal to any number of 'ſquares. As Firſt, to double the ſquare *b d* 'do thus, produce one ſide *b a* making *e a* 'equal to one of the ſides, then join the 'points *d e* and the thing is done, for the 'Square erected on the Line *d e*, is equal to 'the ſquare *b d*, '& alſo the ſquare 'of the Line *a c* '(by the preced-'ing *Theo.*) and 'therefore double 'to either of them.



But if it were required 'to make a ſquare equal to the Summ of 'two ſquares made on the Lines *d e* and *e f*, 'then from *e* erect the perpendicular *e f*, 'and draw the right line *d f*, ſo ſhall the 'ſquare made on the line *d f* be equal to 'the Summ of the Squares made on the lines '*d e* and *e f* by the preceding *Theo.*. Hence alſo

'may the ſubſtraction of Squares be per-
'formed, *i. e.* by it may be found the diffe-
'rence of any two Squares, as if the Square
'made on the line *f e*, were to be taken
'from the Square of *f d*, then by drawing a
'Line equal to *f e*, and from *e* erect a
'perpendicular *d e*, take between your Com-
'paſſes the length of the line *f d*, ſet one
'foot in *f*, and extend the other till it fall on
'the perpendicular at *d*, then join *f d* and
''tis done; for ſince the Square of the Line
'*f d* is equal to the Squares of the Line *f e*,
'*e d*, therefore ſhall the ſquare of the Line
'*d e* be equal to the difference of the ſquares
'of the lines *f d*, *f e*.

'Many other uſeful *Practices Mecanicks*
'perform by this *Theo.* as the finding the
'length of *Strings*, *Hip-Rafters*, *Scaling-*
'*Ladders*, and the like.

THEO. X.

If in a Circle e a c b, *a right line* b d *drawn through the Center biſect any other right line* a c *not drawn through the Center, it ſhall cut it at Right Angles, and if it cut it at Right Angles it ſhall biſect the ſame.*

Dem. Draw from the Center *e* the right lines *e a*, *e c*. Then becauſe *a f* = *f c* (*Hyp.*) and *e a* = *e c* (*def.* 6.) alſo the ſide *e f* common, therefore ſhall the ∠ *e f a* = *e f c* (*Theo.* 2.) and conſequently right (*def.* 33.) *W. W. D.*



Again, becauſe the Angle *e a f* = *e c f* (*Theo.* 3.) and the ſide *e f* common, therefore (*Theo.* 2.) ſhall *a f* = *f c*, that is *a c* is biſected. *W. W. D.*

Cor. 1. Hence therefore the Center of a Circle is in that line, which biſects another at right Angles.

2. Hence in an *Equilateral* or *Iſoſceles* Triangle, if a line drawn from the Vertical Angle biſect the baſe, that line is perpendicular to it and contrary.

Uſe. 'By help of this *Theo.* the Center of 'a Circle may be found, for having applied 'the right line *a c* (*See the preceding Figure*) 'in the Circle *b a c*, and biſected it with the 'perpendicular *b d*, which by the foregoing '*Theo.* will paſs through the Center, and is '(*def.* 8.) the Diameter; If therefore the 'ſaid Diameter be divided into two equal 'parts in *e*, that point of Section ſhall be 'the Center.

'It

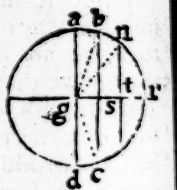
'It also furnishes Artificers with a Geometrical way of finding a Center, whereby 'to strike an Arch when 'they have the height *a b* 'and breadth *c d* given, 'for join the points *b. c*, '*b. d* and produce *b a*, then 'bisect *c b* (or *d b*) with 'the perpendicular *n e*, producing it till it cut *b a* prolonged in *e*, 'which point is the Center that will sweep 'an Arch to the given Height and Breadth.



'*Lastly*, 'Tis used in Trigonometry, for 'by it is proved that the sine of an Arch is 'perpendicular to the Diameter; It also 'helps to demonstrate that the sides of a 'Triangle have the same proportion as the sines of the opposite Angle.

THEO. XI.

In a Circle as g a c b *the greatest Line* a d *is the Diameter, and the next greatest is that which is nearest the Center.*



Dem. Draw the Lines $g b$, $g c$ and $g n$. The Diameter $a d = g b + g c$ (*def.* 6.) but $g b + g c \sqsubset b c$ (*def.* 2.) therefore $a d \sqsubset b c$ *W. W. D.*

Again, $b s^q + g s^q = g b^q$ (*Theo.* 9.) $= g n^q = g t^q + n t^q$. Take away the ſquare of the line $g s$, which is but part of $g t$ from the ſquare of $g b$, there will remain $n t^q$ leſs than $s b^q$, and therefore the ſquare $n t$ is leſs than $s b$, and conſequently $n o$ leſs than $b c$. *W. W. D.*

Uſe. 'This *Theo.* ſhews us that the Chord 'of an Arch greater or leſſer than a Semi-'circle, is leſs than the Diameter.

'*Theodoſius* alſo makes uſe of it to demon-'ſtrate that in a Sphere, the leſſer Circles 'are more remote from the Center than 'thoſe that are greater.

'It is alſo by this *Theo.* that *Ariſtotle* 'proves that the *Rowers* in the middle of a 'Galley have greater ſtrength than thoſe 'that are either at the fore or hinder part 'thereof, for the ſides of the Galley being '*Belging*, the *Oars* of the middle part are 'longer, *i. e.* the *fulciment* or point of 'bearing comes nearer the middle of the 'Oar; and ſuppoſing they pull with equal 'ſtrength, he that ſits in the middle (by 'this property) has his ſtrength increaſed;

'As

'As the Doctrine of the *Mecanicks* in the 'case of the *Ballance* and *Leaver* doth plain-'ly demonstrate.

THEO. XII.

The Angle b a c *at the Circumference of a Circle, is equal to half the Angle* b d c *at the Center, standing on the same (or an equal) Arch* b c *of the Circumference.*

Dem. Firſt, The External $\angle b d c = d a c + d c a$ (*Theo.* 4.) alſo the Line $d c = d a$ (*def.* 6.) therefore the $\angle d a c = d c a$, and conſequently the Angle $b a c$ is equal to half the Angle $b d c$. *W. W. D.*

Secondly, The $\angle b d c$ is double to the $\angle b a c$ by the foregoing caſe, and ſo alſo is the $\angle b d e$ double to the $\angle b a e$, therefore (*ax.* 8.) the $\angle c a e = \frac{1}{2} c d e$. *W. W. D.*



Cor. Hence it follows that the Angles ſubtended by (equal or) the ſame Arch of the Periphery are alſo equal, for that they are both double to the ſame thing, *viz.* the Angle at the Center.

2. That the Angle in a Semicircle is right which (though evident from the forme *Theo.*) may be thus demonstrated. From *d* through the Center draw the line *d e*, then is the $\angle d = \frac{1}{2} a b e$ and the $\angle e d c = \frac{1}{2} e b c$ but half the two Angles *a b e* and *e b c* is a right Angle (*Theo.* 1.) therefore the Angle *a d* qual to one right angle. *W. W. D.*



Schol. *By this* Theo. *it may eaſily be D monſtrated, that Angles made at the Circumference, contain half the quantity of the Arch they ſtand on*, for the Arch *a e b* is the meaſure of the Angle *a c b* (*def.* 13.) but the Angle $a d b = \frac{1}{2} a c b$, therefore the Angle *a d b* equal to half the Arch *a e b*, *W. W. D.*



Uſe. This *Theo.* is uſed in demonſtratin ſome Trigonometrical Propoſitions, as alſ in *Aſtronomy* for finding the *Apogæon* an *Excentricity* of the Sun.

'Alſo in *Opticks* it ſhews 'that the Line *a b* will ap- 'pear of the ſame length 'when ſeen either from *d* 'or *c*, becauſe in both ca- 'ſes you behold it under equal Angles.



Thirdly,

'*Thirdly*, A very easie and practicable 'way is found to make an Angle equal to 'half of any given Angle; as let it be re- 'quired to make an Angle 'equal to half the given An- 'gle *b a d*, on *a* as a Center 'describe with any *Radius* 'the Circle *b c d*, and from 'any point (as *c*) in that 'part of the Circumference,



'not included by the given Angle draw the 'lines *c b*, *c d*, then is the Angle *b a d* dou- 'ble to *b c d* (by the preceding *Theo.*) *i. e.* 'the Angle *b c d* = $\frac{1}{2}$ *b a d*.

'Also the second *Cor.* hereof shews *Me- 'canicks* how to try whether their Squares 'be true, for having described a Semicircle '*a d b*, they apply the Head of their square '*a d b* to the Circumference, then laying 'one side as *d a* upon the extremity of the 'Diameter *a*, the other part or side, *d b* 'will fall exactly on the other extream part 'of the Diameter *b*, if the square be true. 'See the Figure of the said *Cor.*

'Again, by the aforesaid *Cor.* is shewn 'the manner of raising a Perpendicular on 'the end of a given Line, as also of letting 'one fall from a point assigned over the end 'of a given line.

THEO. XIII.

The oppoſite Angles a + c *or* b + d *of every four ſided Figure inſcribed in a Circle, are equal to two right Angles.*



Dem. Draw the Lines *b d*, *a c*, then all the Angles in the Triangle *a b c* are equal to two right, but the ∠ *a d b* = *b c a* (*Cor.* 1. *Theo.* 12.) alſo for the ſame reaſon is the Angle *b d c* = *b a c*, therefore the whole Angle ∠ *a d c* = *b a c* + *b c a* equal to the Complement of the Angle *a b c* to two right, therefore the ∠ *a b c* + *a d c* is equal to two right Angles. *W. W. D.*

Cor. Hence if one ſide of a Quadrilateral Figure inſcrib'd in a Circle be produced, the outward Angle *e b c* is equal to the internal Angle, which is oppoſite to the Angle adjacent.

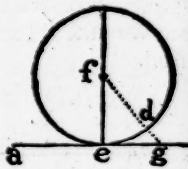
Uſe. 'This Propoſition is helpful in the 'demonſtrating that the ſines of the ſides 'are in the ſame reaſon, as are the ſines of 'the oppoſite Angles; *Ptolomy* alſo makes 'uſe of it to Calculate the Table of Chords.

THEO.

THEO. XIV.

If a right line a b *touch a Circle, and from the point of Contact, through the Center of the Circle, a Diameter be drawn, that line shall be perpendicular to the Tangent* a b.

Dem. If you deny it, let some other line *f g* be drawn from the Center perpendicular to the Tangent, and cuting the Circle in *d*, therefore, whereas the Angle *f g e* is right, (*Con.*) then is the ∠ *f e g* acute, and therefore *f e* (that is *f d*) is greater *f g*, *which is absur'd.*



Schol. *The contrary of this is also Evident, that is, if a right line* a b *touch a Circle, and from the point of contract, another right line* e f *be erected at right angles, it shall pass thro' the Center, that is, it shall be the Diameter of the Circle.*

Use. 'The Tangent or touch line is of great 'use in *Trigonometry*, insomuch that there is 'a Table made thereof, by which in several 'cases we are forc'd to measure the Angles, 'not only of plain, but also of Spherick Tri-

'angles

' angles, alſo in Opticks there are ſome pro
' poſitions founded on touch Lines, pertic
' larly in determining the part of the Glob
' enlightened; the *Theory* of the Phaſes of th
' Moon is Eſtabliſhed on this Doctrine; alſ
' that Celebrated Problem, by which *Hippar*
' *chus* findeth the Diſtance of the Sun by th
' difference of the true and apparent *Quadra*
' *tures*. In Dialling the *Italian* and *Babilonia*
' hours are often Tangent lines, *Maurolic*
' maketh uſe of touch lines, in finding th
' Diameter of the Earth; and in Naviga
' tion we take a Tangent line to be our Hor
' riſon.

Theorems of Proportion.

THEO. XV.

Magnitudes A B, *and* C D, *that have th*
ſame proportion to a third E, F, *have alſo th*
ſame one to another,

Dem.

Dem. For A contains B as oft as E doth F, (*def.* 48.) And C contains D as oft as E doth F, therefore A contains B as oft as C doth D. Or thus $\frac{A}{B} = \frac{E}{F} = \frac{C}{D}$ (*def.* 48.) therefore $\frac{A}{B} = \frac{C}{D}$ and therefore A : B :: C : D. *W. W. D.*

A : B
8 : 6 } E. F
C : D
16 : 8 } 10. 5

Use. 'This way of Arguing is often made 'use of, in several parts of the Mathema-'ticks, particularly in the Solution of *oblique Spherical Triangles.*

THEO. XVI.

If there be several Proportional Magnitudes, viz, *if* A : B :: C : D :: E : F. *then I say there will be the same reason of one antecedent, to its Consequent, as of all the Antecedents, to all the Consequents.*

A : B :: C : D :: E : F.
9 : 3 :: 18 : 6 :: 24 : 8

Dem. Since A is to B, as C to D, A will contain B, as oft as C contains D (*def.* 48.) Add both Antecedents together, as likewise both consequents, then 'tis evident that A contains B, as oft as the Sumn of A and C

contains

contains the Summ of B and D, therefore as A : B :: A + C + E : B + D + F. *W. W. D.*

Cor. Hence if like Proportionals be added to like proportionals, the wholes shall be proportional.

THEO. XVII.

What proportion any two Magnitudes as A *and* B *have one to another, the same proportion shall the like aliquot parts (suppose the fourth) of those Quantities, have one to another, that is as* A *to* B, *(the fourth of* A *) be to* F, *(the fourth of* B. *)*

Dem. Divide A into 4 equal parts as E, H, I, K, and B into 4 equal parts, as F, L, M, N; Now I say, that as E : F :: H : L :: I : M :: K : N for all the antecedents are equal each to other, so likewise are all the consequents. But (*Theo.* 16.) as the Summ of all the Consequents; so is one Antecedent to its consequent, and therefore as A : B :: E : F. *W. W. D.*

A 24 { E 6, H 6, I 6, K 6 } B 8 { F 2, L 2, M 2, N 2 }

Cor. Hence 'tis Evident, that the double, Treble (or any other *Multiplex* whatsoever) of

of two Quantities have the ſame proportion as the Quantities themſelves.

Uſe, 'Theſe two Theorems do many 'times happen in the Mathematical Argu-'ings; they are alſo neceſſary to demonſtrate 'thoſe that follow.'

THEO. XVIII.

If 4 *Magnitudes are proportional*, (viz. *if as* A : B :: C: D) *they ſhall alſo be alternately ſo*, i. e. *as* A : C :: B : D.

Dem. If you deny it, and ſay there is a greater reaſon of A to C than of B to D, A will contain C more times than B doth D, (*def.* 50) let A contain C four times, and B contain D but three times, then the fourth of A will equal to C, but a fourth of B will not be equal to D. Now ſeeing there is the ſame reaſon of A : B :: C : D, there will be the ſame reaſon of $\frac{1}{4}$ A : B as of $\frac{1}{4}$ C : D, (*Theo.* 15.) and therefore if the $\frac{1}{4}$ of A be equal to C, the $\frac{1}{4}$ of B ſhall be equal to D, although the contrary follows if the former ſuppoſition be right, *viz.* that A hath a greater reaſon to C, than B

If A : B :: C : D
41 : 87 :: 12 : 2
then alternately
A : C :: B : D
48 : 12 :: 8 : 2

to

to D, and therefore as A to C, so is B to D. *W. W. D.*

And here Note, That *Alternate* proportion hath place only, when the Quantities are of the same kind, for *Heterogenious* Quantities cannot be compared *Alternately*.

THEO. XIX.

If there be 4 Magnitudes in the same proportion (viz. *if* A : B :: C : D) *they shall also be proportional when inverted.*

Dem. For if A be as great in respect of B, as C is in respect of D, then on the contrary B is as little in respect of A, as D is in respect of C, and therefore as B : A :: D : C. *W. W. D.*

If A : B :: C : D
6 : 2 :: 9 : 3
then inversely
B : A :: D : C
2 : 6 :: 3 : 9

For the quantity of proportion is more generally defined *by how much fold*, rather than by *how many times* the consequent is contained in the antecedent.

THEO.

THEO. XX.

If Magnitudes Compounded be proportional, thy ſhall be proportional when divided, that is if it be as A : D :: B : F *it ſhall alſo be as* A-D : D :: B-F : F.

If A : D :: B : F
16 : 4 :: 8 : 2
then by diviſion
A-D : D : B-F : F
12 : 4 :: 6 :: 2

Dem. Since there is the ſame Reaſon of A to D as of B to F, A will contain D ſo oft as B contains F; now if D be taken from A and E from B once, or any other number of times, then (*ax.* 3.) D is contained in the Remainder A-D, as oft as F is in the Remainder B-F, and therefore (*def.* 48.) as A-D : D :: B-F : F. *W. W. D.*

Schol. *Hence Compounded,* Ratio *is eaſily Demonſtrated.*

For if A-D : D :: B-F : F make each Antecedent to contain his Conſequent once more by adding the reſpective Conſequents to them, then will A : D :: B : F. *W. W. D.*

If A-D : D :: B-F : F
12 : 4 :: 6 : 2
then by compoſition
A : D :: B : F
16 : 4 :: 8 : 2

THEO.

Uſe. 'The forgoing Proportions very fre
'quently occur in ſeveral Mathematical Ar
'guments.

THEO. XXI.

If the whole be to the whole, as the part ta
ken away is to the part taken away, the Re
mainder ſhall be to the Remainder, as the whol
was to the whole, i. e. *If it be as* A : B :: D
F *it ſhall alſo be as* A-D : B-F :: A : B.

Dem. Becauſe A : B :: D : F therefore al
ternately (*Theo.* 18.)
'twill be as A : D :: If A : D :: D : F
B : F and by Diviſion 10 : 6 :: 5 : 3
'twill be as A-D : D :: then 'twill be
B-F : F. And again, A-D : B-F :: A : B
by alternation as A-D :
B-F :: (*Hyp.*) D : F :: A : B therefore
(*Theo.* 15.) A-D : B-F :: A : B. *W. W. D.*

Cor. Hence if like proportionals be taken
from like Proportionals, the Remainder
ſhall be Proportional.

Schol. *Hence is converſe Ratio demonſtrated,* for if A : B :: D : F then alternately as A : D :: B : F but (*by the preceding Theo.*)
then by Converſion as A : A-B :: D : D-F

as

as A : D : : A-B : : D-F, and again by alternation as A : A-B : : D : D.F. *W. W. D.*

After the like way of Arguing may mixt *Ratio* be demonstrated.

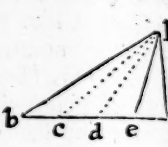
Use. 'This *Theo.* helps to demonstrate the 'Rule of Fellowship, for instead of working by the Rule of Three for every particular *Associate*, having done it for the 'rest, to the last they assign the Remainder 'of the Gain; supposing that if there be the 'same proportion of the whole Summ of all 'the Principals to the whole Gain, as of 'the Principal of one Associate to his part of 'the Gain, there will also be the same proportion of the Principal that remains, to 'the remainder of the Gain.

'Some other ways of arguing there are 'which sometimes are used, but these being the principal and most frequent, I 'omit them.

THEO. XXII.

Triangles b h f, e h f *are in such proportion to each other as their Bases* b f, e f.

Dem. Let $b e = c d = d e = e f$ dra
$b d$, $h c$, then the Tri-



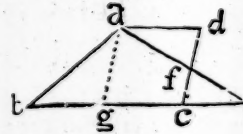
angles $b h c$, $c h d$, $d h e$, $e h f$ are equal (*Theo.* 8.) Now 'tis evident the Triangle $b h f$ contains the Triangle $e h f$, as oft as the base $b f$ contains the base $e f$, and ther fore (*def.* 48.) $\triangle b h f : \triangle e h f$, as the ba $b f : e f$. *W. W. D.*

Schol. *In like manner it may be prov that* pgrms. *of the same or equal height ha such proportion to each other as their Ba for* (Theo. 17.) *as the half is to the ha so is the whole to the whole.*

Use. 'This *Theo.* shews how to cut 'the third part of a *Trapezium* (as $a b c$ 'that hath two of its sides (as $a d$, $b c$) pa 'rallel.

Produce $b c$ and make $c e = a d$, th take $b g$ equal to the third of $b e$ and dra $a g$, I say that the Triangle $a b g$ is one thi of the *Trapezium* a b c d.

Dem. The Tri-



angles $a d f$, $f c e$ are equiangled becauſe of the parallel Lines $a d$, $b e$ and vertical angles, there is alſo in each an equal ſide, *viz.* $a d$,

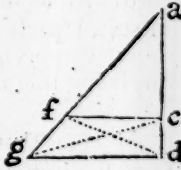
(Co

(*Con.*) thence they are equal to one another (*Schol. Theo.* 2.) and consequently the Triangle *a b e* equal to the *Trapezium a b c d*, but the Triangle *a b g* is the third part of the Triangle *a b e* by this *Theo.* and therefore the Triangle *a b g* is the third part of the *Trapezium a b c d.*

THEO. XXIII.

Similar or Equiangled Triangles a f c, a g d *have their Correſponding ſides proportional.*

Dem. Make them coincident by laying the point *a* on *a*, and the right line *a c* on the right line *a d* &c. then I ſay *f c* is parallel to *g d* (*Schol.* 2. *Theo.* 1.) for the ∠ *a c f* = *a d g*. Draw the lines *c g*, *f d*, then the Triangles *f c g*, *f c d* are equal, being on the ſame baſe *f c*, and between the ſame parallels; But (*Theo.* 22.) △ *a f c* : △ *f g c* :: *a f* : *f g*, alſo △ *a c f* : △ *c d f* (= *f g c*) :: *a c* : *c d*, and therefore (*Theo.* 15.) *a f* : *f g* :: *a c* : *a d*, but (*Theo.* 18.) *a f* : *a c* :: *f g* : *c d*. And (*Theo.* 16.) *a f* : *a c* :: *a f* + *f g* (= *a g*) : *a c* + *c d* (= *a d*) and therefore again (*Theo.* 18.) *a f* : *a g* :: *a c* : *a d*. *W. W. D.*



Schol. *Hence if the ſides of a Triangle be cut proportional, the right line that joins the points of Section, ſhall be parallel to the remaining ſide of the Triangle.*

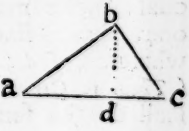
Uſe. 'This *Theo.* is of vaſt Extent, and 'may almoſt paſs for an *Axiom* in all ſorts of 'Meaſuring, it being the very foundation 'both of *Plain* and *Spherical Trigonometry*, 'for tho ſpherical Angles are formed by Ar-'ches of Circles, yet muſt they all be redu-'ced to plain Triangles, before their parts 'can be meaſured: Its alſo from this Pro-'poſition they make and divide ſeveral Ma-'thematical Inſtruments, as *Sinical Qua-'drant*, *Foreſtaff*, *Sector*, *Proportional Compaſſes*, 'with other *Geometrical Inſtruments*. In 'ſhort, 'tis ſo univerſal, that moſt of the 'uſefulleſt parts of the *Mathematicks* are 'founded upon it. Its Uſes and Applicati-'ons being Infinite.

'Alſo upon this *Theo.* depends the practice 'of that *Problem*, which teaches how to cut 'a right line into any number of equal parts. 'See the manner of doing it *Prob.* 10.

THEO. XXIV.

A Perpendicular let fall from the Right Angle of a right angled Triangle a b c, *divideth it into two Similar Triangles* a d b, b d c, *which are also similar to the whole.*

Dem. The ∠ *a b c* = *a d b* because both right, and the ∠ *a* common, therefore the Triangles *a b c*, *a d b* are similar (*Theo.* 4. *Cor.* 3.) By the same Argument are the Triangles *a b c*, *b d c* similar, and therefore (*ax.* 1) the Triangles *a d b*, *b d c* are similar, *W. W. D.*

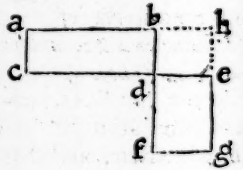


Use. 'This *Theo.* is often used in solveing 'Geometrical Questions by *Algebra*; Also by 'finding the sides of the 5 Regular Platonick 'Bodies inscrib'd in a Sphere. Its also by 'this *Theo.* and the help of a square, an in-'accessible distance as *d a* may be measured; 'for draw the perpendicular *b d*, and set a 'square at the point *b*, so that by looking o-'ver the sides *b a*, *b c* you may see the points '*a* and *c*. Now because by this *Theo. c d*: *d b*:: '*d b*: *d a*, if therefore you multiply *d b* by it 'self, and divide the Product by *c d*, the 'the Quotient shall be *d a* the Line requi-'red.

THEO. XXV.

Equal Equiangled pgrms. *as* a d, d g *have their ſides which are about the equal Angles reciprocal, And equiangular* pgrms. *that have thoſe ſides reciprocal are equal.*

Dem. Let the ſides *c d*, *d e* about the equal Angles make one right line, wherefore *f d*, *d b* (*Theo.* 1. *Cor.* 2.) ſhall do the ſame, and let *a b*, *g e* be produced (*Poſt.* 1.) till they meer in *b*.



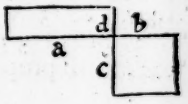
Firſt therefore the *pgrms. a d*, *d g* being equal have the ſame proportion to the *pgrm. b e*, but *pgrm. a d*: *b e*:: baſe *c d*: *d e*, and *pgrm. d g*: *b e*:: baſe *f d*: *d b* (*Schol. Theo.* 22.) therefore (*Theo.* 15.) *c d*: *d e*:: *f d*: *d b*. *W. W. D.*

Secondly, the ſides of the *pgrm.* being reciprocal, there is the ſame proportion of the baſe *c d*: *d e*:: *f d*: *d b*, but as *a d*: *b e*:: baſe *c d*: *d e* (*Theo.* 22.):: *f d*: *d b* (*Hyp.*):: *d g*: *b e* (*Theo.* 21.) therefore (*Theo.* 15.) *pgrm. a d*: *b e*:: *pgrm. d g*: *e b*, and therefore *pgrm. a d* = *d g*. *W. W. D.*

THEO.

THEO. XXVI.

If four lines are proportional, ſuppoſe a : b :: c : d, *the rectangle of the firſt and fourth, ſhall be equal to the rectangle of the ſecond and third. And contrary, If the rectangle of the Extreams and Means are equal, the four Lines are proportional.*



Dem. The Rectangles have each an equal Angle becauſe both right, and *a* : *b* :: *c* : *d* (*Hyp.*) therefore by the laſt part of (*Theo.* 25.) the Rectangles are equal.

Secondly, If they are equal and equiangled, their ſides will be reciprocal, (by the firſt part of *Theo.* 25.) that is, as *a* : *b* :: *e* : *d*. *W. W. D.*

Schol. *In like manner it might be proved, that if three lines are proportional, the ſquare of the middle term ſhall be equal to the Rectangle of the firſt and third.*

Uſe. 'By theſe two Propoſitions is de-'monſtrated the Principal Rules of Com-'mon *Arithmetick*, *viz.* The *Golden Rule*, 'commonly call'd the *Rule of Three*, the 'Rules of *Fellowſhip*, *Alligation*, with o-'thers

thers that depend upon the Doctrine o
Proportion. For ſuppoſe it be
required to find a fourth Pro- *a*, *b*, *c*,
portional to the three numbers 3, 9, 6.
a, *b*, *c*. Then becauſe by this
Propoſition 9 x 6 = 3 multiplied by ano
ther Number unknown. If then I divid
54 by 3 the Quotient is 18, which if mul
tiplied by 3 will give 54 = 9 x 6, an
therefore if 3 give 9, 6 ſhall give 18 fo
the fourth proportional required.

THEO. XXVII.

Equal triangles a c b, e c d *that have i each an equal Angle*, viz. a c b, e c d *have the ſides* (a c, c b *and* e c, c d) *which form the equal Angles reciprocal*; *And contrary, if the Angles are equal and ſides reciprocal, the triangle ſhall be equal.*



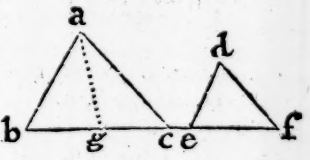
Dem. Let the ſides *a c*, *c d* which are a-bout the equal Angles be ſet in a ſtraight line, then *b c e* is a ſtraight line (*Theo.* 1. *Cor.* 2.) join *a e*.

Firſt,

First, As *a c* : *c d* :: △ *a e c* : △ *c e d* (*Theo.* 22.) :: △ *e a c* : △ *c a b* (becauſe equal) :: *e c* : *c b* (*Theo.* 22.) and therefore as *a c* : *c d* :: *e c* : *c b*. *W. W. D.*

Secondly, △ *a e c* : △ *c e d* :: *a c* : *c d* (*Theo.* 22.) :: *e c* : *c b* (*Hip.*) :: △ *e c a* : △ *c b a* (*Theo.* 22.) therefore (*Theo.* 15.) △ *a e c* : △ *c e d* :: △ *e c a* : △ *c b a*, and therefore (*def.* 48.) △ *c e d* = △ *c b a*. *W. W. D.*

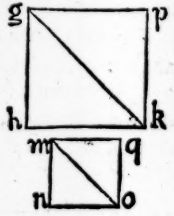
THEO. XXVIII.



Similar triangles a b c, d e f *are in a duplicate Proportion of their Homologous ſides*, that is, *their Area's are as the ſquares of their Homologous ſides.*

Dem. Let *b c* : *e f* :: *e f* : *b g* and let *a g* be drawn. Now becauſe *a b* : *d e* :: *b c* : *e f* :: (*Theo.* 22.) *e f* : *b g* (*Con.*) and the Angle *b* = *e*, therefore is the Triangle *a b g* = *d e f* (*Theo.* 27.) but △ *a b c* : △ *a b g* :: *b c* : *b g* (*Theo.* 22.) Now the proportion of *b c* to *b g* is duplicate to that of *b c* to *e f* (*def.* 55.) therefore △ *a b c* is to △ *a b g* (= *d e f*)

d e f) in duplicate ratio to that of *b c* to *e f*, that is Triangle $\frac{abc}{def} = \frac{bc}{ef}$ twice. *W. W. D.*

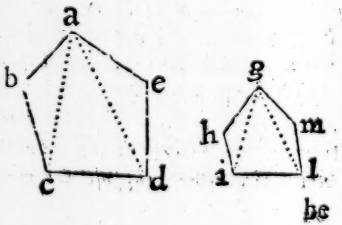


Secondly, that this proportion is as the ſquares of their homologous ſides I thus prove, Let the Triangles be *g h k*, *m n o* (granting all Triangles this property) which ſeverally are half of the Squares *h p*, *n q*. Now ſince the half is to the half as the whole to the whole, the Triangle *g h k* is to the Triangle *m n o*, as the ſquare of *h k* to the ſquare of *n o*. *W. W. D.*

Schol. *Hence 'tis eaſie to prove that all ſimillar ſuperficies are in a duplicate proportion of their Homologous ſides, or as the ſquares of their Homologous ſides.*

Before I paſs to the proof of this Excellent *Theorem*, it's neceſſary to premiſe the following *Lemma.*

Lem. Simillar Polygons *a b c d e*, *g h i l m* may be divided into an equal Number of ſimillar triangles, which triangles will



be like parts of their respective wholes. *First*, As to their Number 'tis evident by Inspection. *Secondly*, That they are like I thus prove, the $\angle b = \angle h$ (*Hyp.*) and the side $ab : bc :: gh : hi$ (*Hyp.*) and therefore if the Triangles are made coincident the line gi will be parallel to ac (*Schol. Theo.* 23.) and consequently the Triangles abc, ghi are simillar. By a like Argument are the Triangles aed, gml simillar. *Lastly*, the $\triangle cad$ like igl, for the whole Figures were equiangled, and equal Angles have been taken away on each side: Therefore the Triangle cad is simillar to igl. *Thirdly*, That they are like parts of their respective wholes. Triangle $\frac{abc}{ghi} = \frac{ab}{gh}$ twice (by the preceding *Theo.*) so also $\frac{acd}{gil} = \frac{cd}{il}$ twice, and $\frac{aed}{gml} = \frac{ed}{ml}$ twice, and because $ab : gh :: cd : il :: ed : ml$, therefore $\frac{ab}{gh} = \frac{cd}{il} = \frac{ed}{ml}$ and therefore Triangle $\frac{abd}{ghi} = \frac{aed}{gml} = \frac{acd}{gil}$ *that is* Triangle $abc : ghi :: aed : gml :: acd : gil$. But (*Theo.* 16.) $\triangle abc : \triangle ghi :: abc + aed + acd : ghi + gml + gil$, and (*Theo.* 18.) $\triangle abc : abc + aed + acd :: \triangle ghi : ghi + gml + gil$, And again $abc + aed + acd : \triangle abc :: ghi + gml + gil : \triangle ghi$, that is, the Polygon $abcde$ is to any one of its Triangles, *suppose* abc, as the other

Po-

of its Triangles, as to the like Triangle *g h i*. This being granted, 'tis eaſie to prove what was propoſed.

For ſince by the third part of the preceding *Lemma* $\triangle abc : \triangle ghi ::$ Pol. *a b c d e*: Pol. *g h i l m*, therefore $\frac{abc}{ghi} = \frac{abcde}{ghilm}$ but $\frac{abc}{ghi} = \frac{ab}{gh}$ twice, therefore *(ax. 1.)* $\frac{abcde}{ghilm} = \frac{ab}{gh}$ twice, And therefore *Pol. a b c d e*: *Pol. g h i l m* is in duplicate Ratio of *a b* : *g h*, or as the ſquare of *a b* to the ſquare of *g h*. *W. W. D.*

Cor. Hence Circles are in a duplicate Ratio to that of their Diameters; for a Circle may be divided into an infinite number of Triangles, as was intimated in (*Theo.* 8.)

2. Hence if 3 right lines are proportional, then as the firſt is to the third, ſo is a Polygon made upon the firſt, to a Polygon made upon the ſecond, like and alike deſcribed, or ſo is a Polygon made upon the ſecond to a Polygon like, and alike deſcribed on the third.

3. If the *Homologous ſides* of like Figures be known, then will the proportion of the Figures be evident, *viz.* by finding a third proportional.

Uſe. 'Hence is diſcover'd a method of 'enlarging or diminiſhing any right lin'd 'Figure in a given Ratio: As if you would 'make

'make a pentagonal Polygon six times as 'big as another, one of whose sides is *a b*, 'then betwixt *a b* and 6 times *a b* find a 'mean proportion, upon which describe a '*Polygon* like the former, and it shall be *Sex-* '*tuple* of the *Hexagon* given, as is evident 'from the preceding Demonstration. The 'contrary of this is also manifest, as sup- 'pose the like sides of two simillar Polygons, 'and the Area of one of them be given to 'find the Area of the other, then because 'their Area's are as the squares of their like 'sides, it will hold as the square of the side 'of the Polygon, whose content is given, 'to the square of the other side, so is the 'content of the given Polygon, to the con- 'tent of the Polygon required. As, suppose '512 be the Area of a superficial Figure, 'one of whose sides is 24, what shall be the 'Homologous side of another like Figure 'whose Area is 768, say as 512 to 768, so 'is 24 to 36, which gives the proportion of 'their Area's, whose first term is the given 'side, then betwixt 24 and 36 find a mean 'proportional, that is, exstract the square 'Root of 24 by 36 it gives the side required.

Lastly, 'It corrects the Error of those 'who are apt to conclude that *Polygons* are 'in such proportion to each other as their 'sides; For if two *Triangles*, two *Squares*, 'two *Polygons*, &c. are proposed, and the 'side of the first double to the side of the

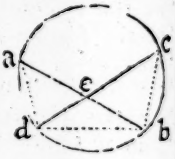
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'ſecond, then ſhall the firſt Figure be fou
'times as big as the ſecond.

THEO. XXIX.

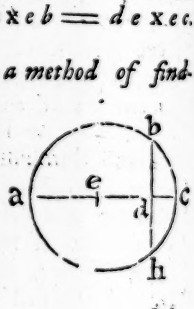
If two right lines a b, c d *interſect one ano-ther in a Circle, the rectangle made of the parts of the one, is equal to the rectangle made of the parts of the other*, that is e a x e b = d e x e c.

Dem. Join the points *d b*, *a d*, *c b*, The



∠ *c e b* = *a e d* becauſe Vertical, alſo the ∠ *a* = *c* (*Cor. Theo.* 12.) and conſequently the ∠ *a d e* = *c b e*, therefore the Triangles *a e d*, *c e b* are ſimillar, and therefore (*Theo.* 23.) *a e* : : *e c* : *d e* : *e b* wherefore (*Theo.* 26.) *e a* x *e b* = *d e* x *e c*. *W. W. D.*

Schol. *Hence is gotten a method of finding a mean proportional betwixt two given lines*; Suppoſe *a d* and *d c*. In an infinite right Line, as *a c*, aſſume the point *d*, and on each ſide it (*poſt.* 3.) cut off *d a* and *d c*, and let through



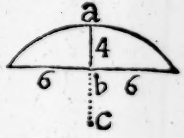
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d the right Line *b h* be drawn at right Angles to *a c*, biſect *a c* in *e*, and deſcribe with the diſtance *e a* or *e c*, the Circle *a b c*, then ſhall the line *d b*, or *d h* cut off by the periphery, be the mean proportional required, It is alſo the ſide of a Square equal to a Rectangle made of the Lines *a d*, *d c*.

Dem. For *d b* = *d h* (*Theo.* 10.) and by the preceding *Theo*, *b d* x *d h* = *a d* x *d c*, that is *b d*^{2} = *a d c* and therefore (*Theo.* 26.) as *a d*: *d b* :: *d b* : *d c*. *W. W. D.*

Uſe. 'This *Theo.* gives Artificers, an A-'rithmetick Rule, for finding the Center to 'ſtrike an arch to any Height and breadth; 'ſuppoſe four Foot high, and twelve Foot 'broad.

Rule, Square half the breadth which is 6, it give 36, divide this by 4, it gives 9, which 9 added to 4 the height, gives 13; half of which is 6 $\frac{1}{2}$, this ſet from the top of the Arch *a*, upon the Line *a b* produced, will give you *c* the Center, upon which you may ſtrike it to the given Dimenſions; ſee the Figure.

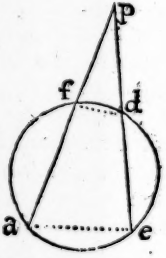


THEO.

THEO. XXX.

If from a point p *without a Circle, you draw two right Lines* p a, p e, *to the oppoſite part of the periphery; the rectangle of one whole Line* p a, *with that part of it* p f, *cut off without the Periphery, ſhall be equal to the rectangle of the other whole Line* p e, *and that part of it* p d, *cut off as before.*

The Triangles *p a e*, *p d f*, are ſimillar for the four ſided Figure *a f d e* being inſcribed, in a Circle makes the Angle *p f d* = *p e a*, (they being both the Complement of the Angle *a f d*, to a Semicircle,) and the Angle *p* is common, therefore are the Triangles *p f d*, *p e a* Simillar, and Conſequently have their correſponding ſides proportional; wherefore as *a p*: *e p* : : *d p*: *f p* therefore (*Theo.* 26.) *a p* x *f p* = *e p* x *d p*. *W. W. D.*



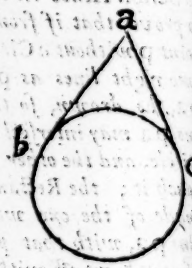
Schol.



Schol. *Hence 'tis eaſie to prove, that if from a point* p *without a Circle, two right lines as* pe, pa, *be drawn, ſo that one* pa *may interſect the Circle, and the other* pe *touch it; the Rectangle made of the one whole line* pa, *with that part of it* pf *cut off without the Periphery, ſhall be equal to the ſquare of the tangent line* pe.

Dem. From *e* draw the Diameter *ed*, join the points *pd* as alſo *ec*, Then are the Triangles *pee*, *ped* ſimillar; For the Angle at *p* is common, alſo the Angle *ped* is right (*Theo.* 14.) and therefore equal Angle *ecd* becauſe in a Semicircle, conſequently the remaining $\angle pec = pde$, and therefore $pc : pe :: pe : pd$, ſo that $pc \times pd = peq$. But by the preceding *Theo.* $pc \times pd = pt \times pa$, therefore (*ax.* 1.) $pf \times pa = peq$. *W. W. D.*

Cor. Hence it appears that two tangents as *a b*, *a c* drawn from the same point without the Circle are equal.

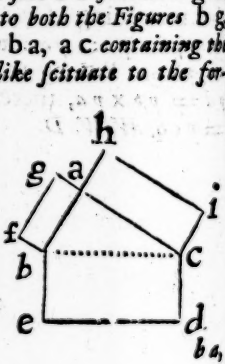


Use. 'The principal use of this *Prop.* 'is to demonstrate the 'third and fourth Axi-'oms in plain Trigo-'nometry.

THEO. XXXI.

In a right angled triangle b a c *any Figure as* b d, *describ'd on the side* b c *subtending the Right Angle, is equal to both the Figures* b g a i *described on the sides* b a, a c *containing the right Angle, like and alike scituate to the former* b d.

Dem. The Figures *f a*, *a i*, *b d*, are alike (*Hyp.*) and therefore (*Schol. Theo.* 28.) in a duplicate Ratio of their Homologous sides; Also by the same



baq, *acq*, *bcq* are in a duplicate Ratio of their *Homologous* sides; Now seeing *baq* + *acq* = *bcq*, therefore shall the Figure *fa* + *ai* = *bd* (*Theo.* 15.) *W. W. D.*

Schol. 1. *Hence a Circle describ'd upon the Hypotenuse of a right angled triangle, is equal to the two Circles describ'd upon the other two sides of the triangle; For* (Schol. Theo. 28.) *Circles are also in a duplicate Ratio to that of their Diameters.*

Note, Such part as *eb* is of *bc*, such part must *ab* be of *bi*, and such part must *ag* be of *gf*. See (*def.* 35.)

Schol. 2. *Hence may be learnt how to Add or Substract any like Figure*, viz. *by using the same method that was follow'd in adding and substracting of Squares in* Theo. 9.

Use. 'This *Theo.* is exceeding useful, it be-'ing an universal Application of the 9 *Theo.* 'to all manner of Figures.

Of Solids.

THEO. XXXII.

One part AC *of a right line cannot be in a plain superficies, and another part* CB *out of it.*



Dem. Produce *A C* to *E*, now if you conceive *C B* to be drawn ſtrait from *A C*, then two right lines *A C B*, *A C E* have one common Segment *A C*, *Which* (*ax.* 8.) *is Abſurd.*



Schol. 1. *Hence any triangle as* D E B *is in one and the ſame plain.* For imagine *E F G* part of the Triangle *D E B* to be in another plain, then *E F* part of the right line *E D* is in one plain, and the other part *F D* elevated above it *Which is Abſurd, by the preceding Theo.*

Schol. 2. *Hence alſo if two right lines as* A B, D C *in the laſt Fig. cut one another, 'tis evident they are in the ſame plain;*

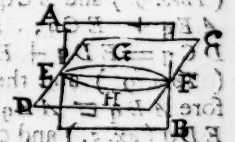
Uſe. 'This Propoſition is of uſe in Dyal-'ling, for by it we prove, that the *ſhade* of 'the *Style* cannot fall out of the plain of 'that great Circle, in which the Sun is.

THEO.

THEO. XXXIII.

If two Plains A B, C D *cut one the other, their common Section* E F *is a ſtrait line.*

Dem. If *E F* be not a ſtraight line, then draw the ſtraight line *EGF* in the Plain *AB*, and the ſtraight line *EHF* in the plain *CD*: Then 'tis evident two ſtrait lines *EGF*, *EHF* include a Superficies, *Which* (*ax.* 9.) *is Abſurd.*

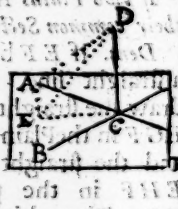


Uſe. 'This Propoſition is of great uſe in 'ſeveral parts of the Mathematicks, but par-'ticularly in *Dyalling*, for the Hour lines on 'all Plains are only the common Section of 'the Hour Circles Plain, with that of the 'Wall.

THEO. XXXIV.

A ſtraight line C D *that is perpendicular to two lines* A C, B C *croſſing, is alſo perpendicular to their Plain* A F.

Dem. If you deny it, let *D E* be perpendicular to the plain *A F*, join the points *E C*, as also *D A*, and at right Angles to *E C* draw *E A*: Now *D E* is perpendicular to *E C* and *E A* (*Theo.* 32. *Schol.* 1. *def.* 68.) also *D C* perpendicular to *C A* (*Hyp.*) therefore $AD^q = AC^q + DC^q$ (*Theo.* 9.) and $AC^q = AE^q + EC^q$, also $DC^q = ED^q + EC^q$ (*Theo.* 9.) and therefore $AD^q \sqsubset AE^q + ED^q$ (*ax.* 5.) and consequently Angle *A E D* cannot be right. The same reason will hold, if the line *D E* touches the plain any where else but in *C*, and so not *D E* but *D C* is perpendicular to *A F* the plain. *W. W. D.*



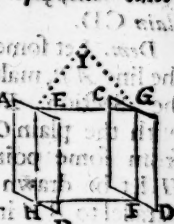
Use. *Theodosius* demonstrates by this propositon, that the Axis of the World is perpendicular to the plain of the Equator; Its likewise used in Dyalling, also in Astralabes, and in cutting of Stones.

THEO. XXXV.

If a plain HG *cut two paralhl plains* A B, C D, *its common Sections* E H, GF

G F *with them shall be parallel lines.*

Dem. For if they be otherwise, being in the same plain, they will if produced meet somewhere, suppose in *I*; Now since the whole lines *HEI*, *FGI* are (*Theo.* 33.) in the plains *A B*, *C D* therefore these plains being produced, shall also meet, *Which is contrary to the Hipothesis.*



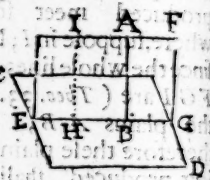
Use. 'This Proposition is of greater use in '*Conick-Sections*, for by it we prove that if 'the *Cone* or *Cylender* be cut by a plain parallel to its base, the Sections are Circles; 'In Dyalling we prove, that the Angles the 'Hour Circles make with a plain parallel to 'a great Circle, are equal to those which 'they make with the Circle it self, likewise in Perspective 'tis proved by this '*Theo.* that the Image of the objective lines 'which are perpendicular to the *Table*, concur in the point of *Vision.*

THEO. XXXVI.

If a right Line A B *be perpendicular to some plain* C D, *all the plains in which that*

right line is, shall be perpendicular to the said plain CD.

Dem. Let some plain as *E F* be drawn by the line *A B*, making the Section *E G*, with the plain *C D*; from some point as *H* let be drawn *H I* parallel to *B A* in the plain *E F*, then shall *H I* be perpendicular to the plain *C D*, because the strait line *H G* makes equal Angles with the parallels *I H*, *A B*, and therefore the plain *E F* is perpendicular to the plain *C D*. *W. W. D.*



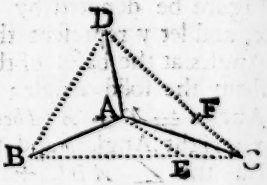
Use. 'The very fundamentals of all Dy-'alling is establish'd on this Proposition, for 'by it we prove that all the Meridians are 'perpendicular to the Equator, since the 'Axis of the World is so, which is their 'common Section; 'tis likewise of great 'use in *Spherick Trigonometry*, *Perspective*, 'and all other parts in which we are obli-'ged to consider plains.

THEO.

THEO. XXXVII.

If a solid Angle ABCD *be contained under three plain Angles* BAD, DAC, BAC *any two of them taken together are greater than the third.*

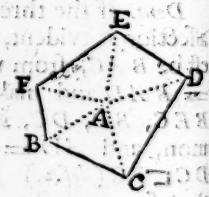
Dem. If the three Angles are equal, the assection is evident, if unequal let the greatest be *BAC*, from whence take away *BAE* = *BAD* and make *AD* = *AE* drawing *BEC*, *BD*, *DC*, Now the side *BA* is common, and *AD* = *AE* (*Con.*) Also *BD* + *DC* ⊏ *BC* (*def.*2.) therefore rejecting *BD* and *BE* in both, there shall remain *DC* ⊏ *EC*. But since *AD* = *AE* and *AC* common, also *DC* ⊏ *FC* the Angle *CAD* shall be greater than *EAC* (for if *E* be turned over *AC* it will fall but at *F*) Therefore the Angle *BAD* + *CAD* ⊏ *BAC*, *W.W.D.*



THEO. XXXVIII.

Every Solid Angle A *is contained under less than* 4 *right Angles.*

Dem. Let a plain cut off the top of the solid Angle, and make the many sided Figure *B C D E F*, which from some point in the Figure let be cut into Triangles, then let the Summ of the Angles *B, C, D, E, F*, of the said Figure be denoted by x, and let y represent the Summ of all the Angles at the bases of the Triangles which form the solid Angle; Then $x + 4$ right Angles $= y + A$ (*Theo.* 4.) for $y + A =$ 10 right Angles, and so is $x + 4$ right. But the $\angle ABC + ABF \sqsubset CBF$ (*Theo.* 37.) the same holds in the Angles *C. D. E. F.* And therefore 'tis evident that $y \sqsubset X$, and consequently that *A* which denotes the solid Angles, is less than 4 right. *W. W. D.*



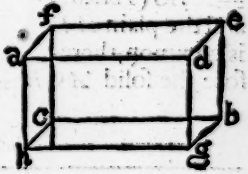
Use. 'These two Propositions are made 'use of to demonstrate some that follow, as 'also when of several plain Angles there 'may

'may be made a solid Angle, the know-
'ledge of which is very requisite, in the
'cutting several Bodies in Wood, Stone, &c.

THEO. XXXIX.

The opposite Plains a c, d b *of a Parallelepipedon are alike and equal.*

Dem. The plains *a c*, *d b* are parallel (*def.* 89.) therefore (*Theo.* 35.) the intersections *a f*, *d e* are parallel as are also *e f*, *d a* and therefore *a e* is a *pgrm.* & by consequence *a f* = *d e*. In like manner it might be prov'd that all the other plains are *parallelograms*,

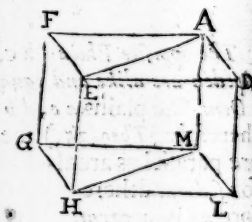


and that the sides in one *pgrm.* are respectively equal to the sides in its opposite: Therefore the Angle *f a b* = *e d g* and *a f c* = *d e h* and therefore the plain *a c* is equal and like to the plain *d b*. *W. W. D.*

THEO. XL.

A Parallelepipedon F L *is divided into tw*
equal parts, by the Diagonal plain H A.

Dem. All the oppoſite *parallelograms* are equal (*Theo.* 39.) And the Triangle *AED* = *AEF*, alſo the Triangle *L M H* = *MHG* (*Theo.*6.) and the plain *A H* is common, therefore the ſolid *AGH* = *HDA*. *W. W. D*

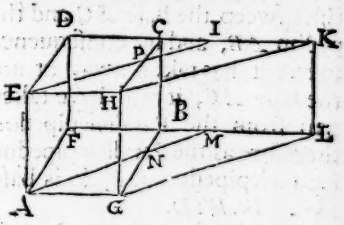


THEO. XLI.

Parallelepipedons A D *and* C K *are equal*
which have the ſame or an equal baſe, and
the ſame or an equal height.

Dem

Dem. The Prisms *AFMEDI, GBLHCK* are equal (*def.* 74.) from both these take away the common prism *NBMPCI*, and add to the remainders the solid *AGNEHP* it makes the parallelepipedon *AC* equal to the parallelepipedon *CK*. *W. W. D.*

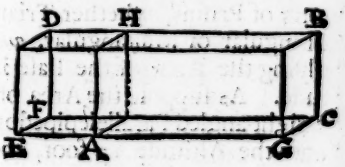


This Proposition may more universally be prov'd (that is for all sorts of Prisms) by Infinites after the method of (*Theo.* 7.)

THEO. XLII.

Parallelepipedons AD, AB *of the same or equal height, are one to another as their Bases.*

Dem. Let the base *AF* be conceiv'd produced to *GC*, and the plain



AH

AH which is parallel to *ED* (*def.* 89.) b
mov'd parallelly to *GB*, it ſhall at the ſam
time ſweep the Baſe *AC* and the Parallelep
pedon *AB*, and by conſequence how muc
ſoever it has taken away at any time fro
the Baſe *AC*, it will have taken away a li
part from the Parallelepipedon *AB*; an
therefore as the Parallelepipedon *AD* to th
Parallelepipedon *AB*, ſo is baſe *AF* to ba
AC. *W. W. D.*

What has been demonſtrated of Parallel
pipedons in the 41 and 42 Propoſition
holds alſo in triangular Priſms, they bein
half thereof, by (*Theo.* 40.) And there
fore

Cor. 1. Triangular Priſms which hav
the ſame Baſe and Height are equal.

2. If They have the ſame or an equ
height, they are in ſuch proportion as thei
Baſes.

3. Theſe properties do likewiſe hold i
all polygonous Priſms, as alſo in the Cylen
der it ſelf, which is conſtituted by an Infi
nite number of Triangular Priſms.

Uſe. 'From the three preceding Theo
'rems is learnt the way of meaſuring al
'ſorts of Priſms, whether Triangular, Qua
'drangular or Multangular, *viz.* by multi
'plying the Area of the Baſe into the Alti
'tude. As ſuppoſe the Area of the Baſe o
'a right angled Parallelepipedon be 48 foot
'and the Altitude 17 foot, then multiply
'ing

'ing 48 by 17, the product 816 is the so-
'lid content of the said Figure in Cubick
'Feet.

'Moreover, Since the whole Paralellepi-
'pedon is produced of the Altitude multi-
'plied into the Base, the half thereof that
'is a Triangular Prism shall be produc'd
'of the Altitude drawn into half the Base.
'Namely the Triangle.

'From this also the solidity of all polygo-
'nous Prisms, as also of the Cylender is
'foun'd, *viz.* by multiplying the Area of
'the Base into the height.

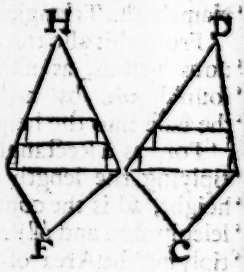
'For, as a Rectangle is produced by mul-
'tiplying the length of the Base into the
'height, so is the content of a right Paral-
'lelepipedon and all Prisms produc'd, by mul-
'tiplying the Area of the Base by the Alti-
'tude; And therefore (*Theo.* 41.) the solidity
'of all Prisms is found by multiplying the
'Base by the height.

THEO. XLIII.

All Pyramids as CD, FH *having the same Altitude and equal Bases are equal.*

Dem. If you deny it let *CD* be greater than *FH*, and let there be inscrib'd in this Pyramid any number of Prisms having like Bases

Baſes and equal heights, whoſe ſolidity may be greater than the Pyramid *F H*; then theſe Priſms ſo inſcrib'd, will be leſs than the Pyramid in which they are inſcrib'd; for the leſſer heights the Priſms have, the more will there be in number, *that is* by leſſening their height you may encreaſe their number, ſo that it ſhall come nearer to the Pyramid *C D*, than any other Quantity, ſuppoſe *F H*. Let there be alſo inſcrib'd in the Pyramid *F H* the ſame number of Priſms as was in the Pyramid *C D*.



I ſay the Priſms in each of theſe Pyramids are equal, for their number is equal by *Conſtruction*, their Baſes and Heights may be alſo equal, becauſe thoſe of the Pyramids in which they are inſcrib'd, are ſo (*Hyp.*) *that is*, all the Priſms in *H F* is equal to all the Priſm in *C D*, which by conſtruction was made greater than the Pyramid *F H*, *that is* a part is equal to the whole, *Which is Impoſſible*; And therefore the Pyramid *C D* = *F H*. *W. W. D.*

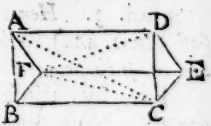
Schol

Schol. *Hence all Cones are equal, which have an equal height and base; for they may be divided after the same manner, and therefore the like Argument will hold.*

THEO. XLIV.

Every Prism as ABCDEF *having a Triangle for its Base, may be divided into three equal Pyramids* FDEC, CBFA, FADC.

Dem. Draw Diagonals FC, FD, AC which shall bisect the *pgrm* (*Theo.* 6.) Now the Pyramid FDEC = *pyram.* CBFA because they have equal bases and heights; Also for the same reason *pyram.* FACB = *pyram.* FADC, therefore (*ax.* 1.) the three Pyramids FDEC, CBFA, FADC are equal. *W. W. D.*



If you cut this Figure out in Cork, or the like, and then consider it, the *Dem.* will be very clear.

Schol. *Hence 'tis easie to prove that every Pyramid is a third part of the Prism, that has the same base and Altitude with it; Or every Prism is treble of the Pyramid that has the same base and Altitude.*

For conceive any poligonous Prism a
Pyramid whose bases are equal and like a
height the same; Divide the first into tria
gular Prisms, and the other into triangular P
ramids, then all the parts of the Prism shall
treble to all the parts of the Pyramid, by t
preceding *Theo.* and consequently the wh
Prism is treble to the whole Pyramid. *W.W.*

Schol. 2. *Hence also we may reasona*
conclude, that a Cone is the third part o
Cylender, that has the same base and Altit
with it. For the Cylender may be divid
into an infinite number of triangular Prisi
and the Cone into a like infinite number
triangular Pyramids.

Schol. 3. *Hence, and from the Corollar*
(*of* Theo. 42.) *Pyramids of the same heig*
are in such proportion as their Bases;

For by those Corollaries all Prisi
are so. Now Pyramids of the same b
and height are the third of those Prism
by the preceding Theo. therefore in such pr
portion as their Bases: For as the whole
to the whole, so is any part to the like pa
This property will also hold in Cones.

Use. 'Upon this *Theorem* is founded t
'reason of measuring all Pyramids, wheth
'Triangular, Rectangular or Multangula
'as also of the Cone or Sphere; For fin
'(*Schol.* 1.) of the preceding *Theo.* every P
'ramid is $\frac{1}{3}$ of the Prism that has the sam
'base and height with it, therefore the so

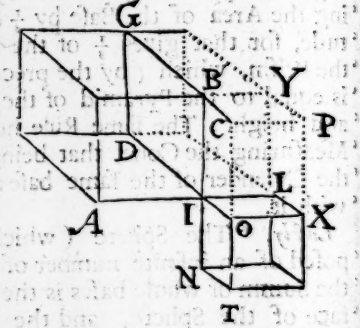
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'dity of all pyramids, is found by multiply-
'ing the Area of the Base by $\frac{1}{3}$ of the Alti-
'tude, for that gives $\frac{1}{3}$ of the solidity of
'the Prism, which (by the preceed. *Theo.*)
'is equal to the Pyramid of the same base
'and height; The same Rule holds also in
'Measuring the Cone, that being the $\frac{1}{3}$ of
'the Cylender of the same base and height
'with it.

Lastly, 'The Sphere (which is com-
'posed of an infinite number of Pyramids,
'the Summ of whose bases is the whole sur-
'face of the Sphere, and the height the
'Radius of the Sphere) is measured by mul-
'tiplying the Area of the surface by $\frac{1}{3}$ of its
'Semi-diameter.

THEO. XLV.

Simillar Parallelepipedons AG, NX *are in a triplicate proportion of their Homologous sides, that is, their solidities are as the Cubes of their Homologous sides.*



Dem. Let the Parallelepipedons be A G, N X. Fill up the void ſpace with the Parallelepipedons D Y, I P, then ſhall the ſide A I: I L:: D I: I O:: B I: I N *(Hyp.)* alſo Parallelogram A D: D L :: D L: I X:: B O: I T: *(Schol. Theo. 22.)* that is, the Parallelepipedon A G: D Y:: D Y: I P:: I P: N X (*Theo.* 42.) therefore the proportion of A G: N X is triple of A G: D Y *that is* of A I: I L their Homologous ſides. *W. W. D.*

Secondly, That this Proportion is as the Cubes of their Homologous ſides I thus prove, All Cubes are *like Parallelepipedons*, and conſequently are in a triplicate proportion of their Homologous ſides *by the preced. Theo.* But like Parallelepipedons are in the ſame reaſon as Cubes; for upon two given lines conceive two ſimillar Parallelepipedons made, and upon the ſame lines two Cubes, then becauſe the Parallelepipedons are in a triplicate Ratio of thoſe given Lines, and alſo the Cubes in a triplicate Ratio of the

ſame

ſame Lines, therefore the Parallelepipedons are one to another as the Cubes of their Homologous ſides.

Cor. 1. What has been prov'd in the preceding *Theo.* concerning like Parallelepipedons, holds alſo in like triangular Priſms as being their halves (*Theo.* 40.) As alſo in all Poligonous Priſms, of which the Triangular is an Aliquot part. And conſequently in the Cylender, which is made up of an infinite number of triangular Priſms; So that the ſolidity of all Cylenders are as the Cubes of their Diameters. The ſame property holds alſo in all Pyramids and Cones, which are the third part of their Circumſcribing Priſms and Cylenders.

Schol. *Hence 'tis eaſie to prove that all ſimillar Solids are in a triplicate proportion of their Homologous ſides.*

All like Solids, whether they be Regular or Irregular, may be reſolv'd into ſimillar Pyramids: For let there be two ſimillar Solids, in one of which conceive a point, from which Imagine ſtraight lines drawn to all the Angles of one of the Figures, divided into Pyramids, whoſe number are equal to the number of Plains that bound the ſolid; do the like in the other ſolid, *that is*, conceive a point poſited in all reſpects here as it was in the other, *that is*, ſo that the ſides of any one of theſe Pyramids in one of theſe Solids; be proportional to the like ſides

of the like Pyramid in the other solid, then 'tis naturally evident that these two solids are divided into an equal number of similar Pyramids, which are like parts of their respective wholes. Now since these Pyramids (which are like aliquot parts) are in a triplicate proportion of their sides, and these aliquot parts in such proportion as their wholes (*Cor. Theo.* 17.) therefore the whole solids must be to each other in a triplicate proportion of their Homologous sides.

Cor. Hence 'tis evident that Spheres are in triplicate proportion of that of their Diameters; For a Sphere is composed of an infinite number of like Pyramids, whose bases are part of the surface of the Sphere, Tops the Center of the Sphere, and heights the Radius of the Sphere. Now these Pyramids are to each other as the Cubes of their heights, which is the Semidiameter of the Sphere, and which I take for one of their sides, as differing insensibly therefrom; But these Pyramids being like parts of their wholes, therefore the whole Spheres are in a triplicate proportion of that of their Semidiameters; Now the Semidiameters are to each other as their whole Diameters; therefore Spheres are in a triplicate proportion of that of their Diameters.

Use. 'Upon this *Theorem* is grounded 'many excellent Properties, particularly the 'augmenting or encreasing Solids in a given

'Ratio;

'Ratio as suppose I have a Cube whose side 'is 7 Inches, and I would find the side of ano-'ther Cube that is double to it: Here 'tis 'evident that the side of this second Cube 'cannot be double the side of the first, for if 'so, then like Solids were in such propor-'tions as their sides, which the preceding '*Theo.* proves otherwise, *that is*, that they 'are in a triplicate Ratio of their Homolo-'gous sides; And therefore, to find to the 'side of this double Cube, 'tis only to find 'two mean proportionals betwixt the side 'of the Cube propos'd and double its side; 'Hence 'tis easie to solve that Celebrated 'Proposition of the duplication of the Cube 'propos'd by the Oracle.

'Also by this Proposition we correct the 'false opinion of those which imagine that 'like Solids are in the same Ratio as are their 'sides; *that is*, that a Cube whose side is a 'foot, is but half of the solid whose side is 'two foot. Also in Naval Architecture, if 'you would keep or observe the same pro-'portion: For Example, If the Keel of a 'Ship of 150 Tunn be 54 foot long, what 'length ought the Keel of that Ship to be 'whose Burthen is 350 Tunn. Say first, If '150 give 360, what shall 54 give, the 'answer is 126, the proportion of the Soli-'dities, whose first term is the given side; 'Find therefore two mean proportionals be-'twixt 54 and 126, by extracting the Cube

'Root of the Product of the Square of 54 in-
'to 126, *that is* of 367416, and it gives you
'72 *prope* the Keel of the Ship required; Ha-
'ving thus gotten the length of the Keel of a
'Ship of this Burthen, the Dimensions of all the
'other Members that compose the Ship are
'easily acquir'd by a single Proportion thus;
'As 54 foot to 72 foot, so is 36 foot the length
'of a Beam in the first, to 48 the length of
'the Beam in the second; For 'tis supposed
'that the Dimensions of all the Members
'and Parts of any one Vessel be known.

THEO. XLVI.

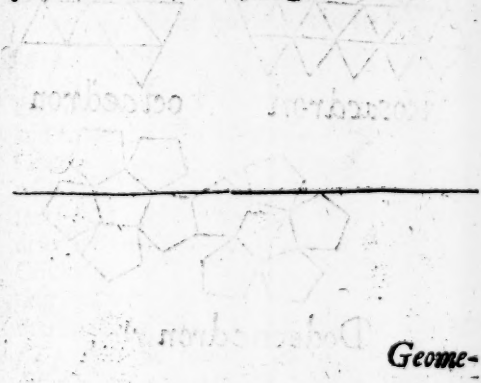
There can be but 5 Regular Bodies, that is, *such that have all their sides, and all their Angles equal.*

Dem. There is no plain Figure when join'd together can make a solid Angles, except a Triangle Square and Pentagon: For a solid Angle (*Theo.* 38.) ought to consist of less than 4 right Angles, and three plain Angles are the fewest that can make up a solid one (as is naturally evident) Now three Angles of a *Hexagon* are equal to 4 right, and in all Figures of more sides than a *Hexagon* they exceed 4 right. Wherefore 3, 4 and 5 Triangles (for 6 Angles of an Equi-

lateral

lateral Triangle make 4 right) 3 Squares, and 3 Pentagons (which make $\frac{2}{10}$ of 4 right Angles) may all compose a solid Angle; And therefore there can be but 5 Regular Bodies, *viz.* A *Tetraedron*, whose Angle is made up of 3 Equilateral Triangles; A *Cube*, the Angle of which is formed by 3 Squares; An *Octaedron*, whose Angle is made up of 4 Equilateral Triangles; A *Dodecaedron*, whose Angle is formed by 3 Pentagons; And an *Icosaedron*, whose Angle consists of 5 Equilateral Triangles.

If you cut out the following Figures in Pastboard and fold them up, they will represent the aforesaid 5 Regular Bodies.



Geome-



cube



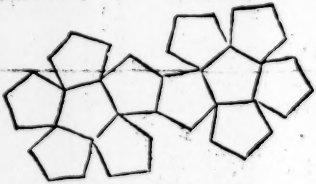
tetraedron



icosaedron



octaedron



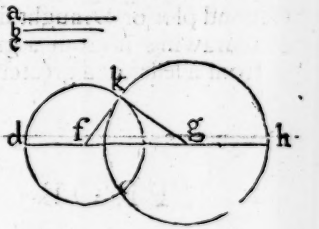
Dodecaedron

Geometrical Problems.

PROB. I.

HOW *of* 3 *ſtrait lines* a b c *to make a Triangle. Of which it is neceſſary that any two of them taken together be longer than the third.*

On the Infinite line *d h* take (*poſt.* 3.) *d f*, *f g*, *g h* equal to the given lines *c*, *a*, *b*, and from the Centers *f g*, with the Extents *f d*, *g h* deſcribe two Circles cutting each other in *k*; draw *f k*, *k g* and the Triangle *f k g* ſhall be made, whoſe ſides (*poſt* 3. *def.* 6.) are equal to the 3 right lines given *W. W. to be done.*



Schol. *In like manner may be deſcribed either an* Equilateral *or* Iſoſceles *Triangle, if ſo be*

be that for the first, the 3 Circles be equal, and for the later two of them.

Use. 'Upon this *Prob.* depends the me-
'thod, not only of making one Figure equal
'to another, but also of enlarging or dimi-
'nishing the same. For having divided the
'Figure into Triangles, measure each side,
'and set down each sides measure upon it
'self, then according as you would have the
'Figure, either equal, greater or lesser, take
'from a Scale (that is equal greater or lesser)
'the number of parts that each of these sides
'contain, and with these Extents make Tri-
'angles, as in the preceding *Problem* is
'taught.

'So that upon this Proposition, the busi-
'ness of Enlarging or Diminishing any
'Ground plot or Draught depends; *that is,*
'of redrawing it from a greater to a lesser,
'or from a lesser to a greater Plan.

PROB. II.

From any given Right line as e d, *and point* c *therein assign'd, to raise a Perpendicular.*

On each side the given point *c*, cut off (*post.* 3.) *c d* = *c e* with the distance *d e* (or any other not less than *d c*) from the points *d e*, describe two Arches intersecting each other in *f*, then join *f c*, and it is the perpendicular required. The *Dem.* depends on *Theo.* 2.



PROB. III.

If the Perpendicular had been required to be raised in or near the end of the Line d e, *suppose the point* e.

With any small distance set one foot of the Compass in *e*, and extend the other (any where on that side the line *d e*, the Perpendicular is to be raised as) to *b*, on *b* as a Center describe with the space *b e* the Semicircle *a e f*, drawing from *a* (the point where the Arch cuts the line *d e*) thro' the Center *b*, the Diameter *a b f*, then join the points *f e*, it gives the Perpendicular required. The *Dem.* depends upon *Cor.* 2. *Theo.* 12. where 'tis prov'd



prov'd that an Angle in a Semicircle
right.

Use. 'Theſe *Problems* are of great u
'with moſt Mechanick Artificers; Th
'drawing of Perpendiculars being very fr
'quently required.

PROB. IV.

To divide a Right lin'd Angle c *into tw*
equal parts.

Set one foot of the Compaſſes in *c*, an
ſtrike the Arch *a d*, this
cuts from each leg an equal
part, on the points *a* and *d*
with the ſame or any other
diſtance, deſcribe the prickt
Arches interſecting each o-
ther in *e*, then join *c e* and
and it performs what was
required: For the Angle
$\phi c e = e c d$. The *Demo.* depends upo
Theo. 2.



Cor. Hence it appears how an Angl
may be cut into 4. 8. 16. *&c.* equal part
viz. by a continual biſection of each par
But the manner of cutting Angles into an
number of equal parts, is as yet unknow
to *Geometricians.*

Uſ

Use. 'This *Problem* is very useful in dividing the Limbs of several Instruments, as '*Theodelites*, *Semicircles*, *Quadrants*, and the 'like, since 'tis the same thing to divide an 'Arch as an Angle.

PROB. V.

To cut a Right line a d *into two equal parts.*



On the ends *a* and *d* of the given right line, describe with any distance two Arches as *c a f*, *c d f* join the intersections *c*, *f* with the right line *c f*, and it shall divide the given line *a d* equally in *e*. The *Dem.* depends upon *Theo.* 2.

Use. 'This *Prob.* is very necessary for the 'dividing of Measures, and making of 'Scales.

PROB. VI.

From a point above as c, *to let fall a Perpendicular to a given line* a b.

From

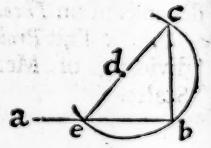
From *c* with any distance greater than *c e*, deſcribe an Arch of a Circle, cutting the given right line in the Points *a* and *b*, then biſect *a b* in *e*, (*Prob.* 5.) and draw the line *c e*, and the thing is done. The *Dem.* depends upon *Theo.* 2.



PROB. VII.

From a point c *that is over the end of a given line as* a b, *to let fall a Perpendicular.*

From *c* to any point as *e* in the line *a b* draw the ſtrait line *c e*, which biſect in *d*, on *d* as a Center with the diſtance *d c* or *d e* deſcribe the Semicircle *e b c*, and join *c b*, ſo is the right line *c b* the Perpendicular required. 'Tis *Dem.* after the ſame manner as was *Prob.* 3.



Uſe. 'Theſe *Problems* are of great and 'general uſe by all ſorts of Artificers; Alſo in

'in *Surveying*, *Fortification*, *Dyalling*, and ſe-'veral other parts of the Mathematicks.

PROB. VIII.

To draw a right line a b, *parallel to a given right line* c d, *and through* b, *a point aſſigned.*

This caſe is univerſal, becauſe all others fall under it, and the practice and Demonſtration is ſhown in the uſe of (*Theo. 6.*)

Uſe. 'Parallel lines are of great uſe in 'moſt parts of the Mathematicks, particu-'larly in *Perſpective*, *Navigation*, *Surveying* 'and *Dyalling*.

PROB. IX.

To make a right lin'd Angle b, *equal to a right lin'd Angle given* a.

Draw the Right Line *b f*, ſet one foo
the Compaſſes in the point *a*, and at any convenient diſtance deſcribe the Arch *c d*, alſo with the ſame extent ſet one foot in *b*, and deſcribe the Arch *e f*. This done take between your Compaſſes the diſtance *c d*, and ſet from *f* to *e* in the other Arch, draw *b e* it forms the Angle required, Demonſtration of which depends u (*Theo.* 2.)



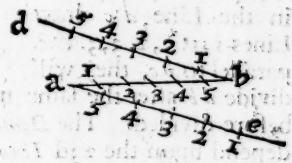
'This *Prob.* is ſo abſolutely neceſſary t 'without the knowledge of the prac 'hereof, nothing could be perform'd eit 'in *Surveying*, *Fortification*, *Perſpective*, *alling*, *Drawing*, &c.

PROB. X.

To divide a Line a b *into any number equal Parts.*

Suppoſe it be required to divide the Li *a b* into 6 equal parts; draw from *b* t Line *b d*, making an Angle at pleaſure wi the Line *a b*, alſo from *a* the contrary w and ſide draw the line *a e*, making the ſam wi

with Angle *a b*, as the former Line *b d* did; then run any small distance 5 times from *b* on the line *b d*, which must always be one less than the number you intend to divide the Line into, do the like with the same space on the Line *a e*, but the contrary way. Then lay a Ruler from the Figure 5 in each Line, it shall cut the Line *a b* in 1. Proceed thus, by laying the Ruler over 4, 4. 3, 3, *&c.* and it will divide the Line *a b* into 6 equal parts as was required.



The Reason and Demonstration of the practice of this *Problem* is grounded upon the 23d *Theo.*

PROB. XI.

To divide a Line d b *after the same manner as another given Line* d c *is divided.*

Make with the two Lines *d b*, *d c* the Angle *b d c* at pleaſure, join the points *b c*, and from the points 5. 4. 3. 2. 1. in the Line *d c* draw Lines 1, 1. 2, 2. 3, 3. *&c.* parallel to *b c*, they will divide *d b* after the ſame manner as *d c* before divided. The *Dem.* of this does depend upon the 23d *Theo.*

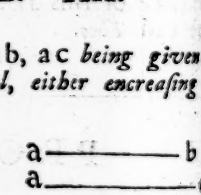


Uſe. 'Both this and the former *Prob* 'neceſſary for the diviſion of Inſtrument

PROB. XII.

Two Right Lines a b, a c *being given find a third proportional, either encreaſing diminiſhing.*

Make with the two given Lines an Angle at pleaſure, as *b a c*, join *b c*; then (if you would have the proportion encreaſing) produce *a b*, ſo that *a e* = *a c*, from *e* parallel to *b c* draw *e d* producing



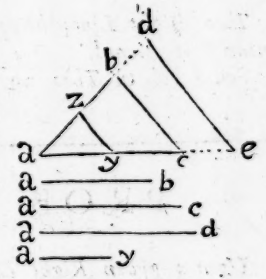
a c till it meet in the point *d*, ſo is *a d* the fourth proportional encreaſing; for as *a b* : *a c* :: *a e* (*a c*) : *a d*. *Theo.* 23.

If you would have it diminiſhing, then make *a f* = *a b*, and from *f* draw *f g* parallel to *c b*, ſo is *a g* the fourth proportional diminiſhing; for as *a c* : *a b* :: *a f* (*a b*) : *a g*. *Theo.* 23.

PROB. XIII.

Three Right Lines a b, a c, a d *being given, to find a fourth proportional, either Encreaſing or Diminiſhing.*

If you would find it Encreaſing, make an Angle at pleaſure with the two ſhorteſt Lines *a b*, *a c*, produce *a b* till *a b d* = *a d* the longeſt of the three given Lines; from *d* draw *d e* parallel to *b c*, producing *a c* till it meet it in *e*, ſo is *a e* the fourth proportional: For as *a b* : *a c* :: *a d* : *a e*. *Theo.* 23.



If you would find it diminishing, let *a c* be the two longest of the three giv Lines, and *a y* the shortest, then having fo med an Angle with the two longest of the cut from *a c*, *a y* equal to the least of t three given Lines, from *y* draw *y z* par let to *b c*, so is *a z* the fourth proportio Diminishing; for as *a c* : *a b* :: *a y* : *a Theo.* 23.

Use. 'These *Problems* are very usef 'particularly in dividing the Sector a 'proportional Compasses.

PROB. XIV.

Two Right Lines being given, to fin mean Proportional.

See *Schol.* of *Theo.* 29.

PROB. XV.

Upon a given Right Line a d, *to mak Geometrical Square.*

Fr

Erect from *d* the Perpendicular *d c* = *a d*, on the points *c*, *a* describe two Arches intersecting one another in *b*, join *a b*, *c b* and the thing is done. The *Dem.* is clear, for one Angle being right, and the 4 sides equal by *Construction*, the rest of the Angles are right; wherefore (*def.* 24.) the Figure *a b c* is a true Geometrick ſquare.

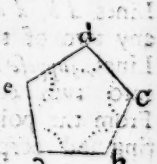


By the ſame method may the Rectangle or long Square be deſcrib'd, if the diſtance *d c* be taken greater or leſſer than *a d*.

PROB. XV.

Upon a given Line a b, *to make a Regular or Equilateral Pentagon* a b c d e.

Find the Degrees contained in one Angle, which (*Theo.* 4. *Schol.* 2.) are 108 *deg.* on the point *b*, make (*Prob.* 9.) an Angle of 108 *deg.* with the given line *a b*, and take *b c* = *a b*, on the point *c* make another Angle of 108 *deg.* and make *c d* = *b c*; pro-



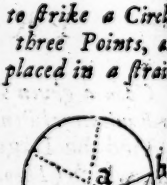
 ceed

ceed thus with the reſt of the Angles an
ſides till you come to the Angle *e*, wher
you need but join the points *e a* and the Fi
gure is compleated; For the Angles *e* and
are eaſily prov'd equal to the reſt; Where
fore (*def.* 31.) the Figure *a b c d e* is a regu
lar *Pentagon.*

In like manner may any other regula
Polygon be deſcribed upon a given Line, b
laying down the Angle as found (*Theo.*
Schol. 2.) and ſo encloſing the Figure wit
equal ſides.

PROB. XVII.

*To find a Center whereby to ſtrike a Circl
that ſhall paſs through any three Points, a
b c d, Provided they are not placed in a ſtrai
Line.*



Join the three points together by the ſtraight Lines *d b*, *d c*, *b c*, cut any two of thoſe three Lines, *ſuppoſe d c* and *b c*, into two equal parts. From the points of cutting and perpendicular thereto, draw the right Lines *e a*, *g a*; the point *a* where theſe Lines cut each other is the Center of the

the Circle; From which if with the distance *a b*, *a c* or *a d* you describe a Circle, it shall pass through the three given points *b. c. d.* as was required. The *Dem.* depends on *Theo.* 16.

PROB. XVIII.

To make a Square equal to a given Rectangle.

This is easily deducible from the 29th *Theo.* for the side of a Square equal to any given Rectangle, is only a mean proportional betwixt the longest and shortest side of it. *See also the* 14*th* Prop. *of* Euclids *Second Book.*

PROB. XIX.

To make a Square equal to the Sum or difference of any two Squares given.

See the Practice and Demonstration of it, in the use of *Theo.* 9.

PROB.

PROB. XX.

To divide a Parallelogram into two equal parts, by a Line that ſhall paſs through any aſſigned point within the Figure.

See the Practice and *Dem.* of this *Problem* in the uſe of *Theo.* 6.

PROB. XXI.

To divide a Triangle into any number of equal Parts.

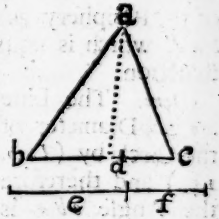
See the Practice and *Dem.* of this in the uſe of *Theo.* 8..

PROB. XXII.

To divide a Triangle a b c *into two parts by a Line drawn from an Angle* a, *in ſuch proportion as the Line* e *to the Line* f.

Divide

Divide the ſide *b c* (*Prob.* 11.) in ſuch proportion as *e* to *f*. From *a* draw the Line *a d* and it is done; for as *e* : *f* :: *b d* : *d c* (*Con.*) : △ *b a d* : △ *d a c*. *Theo.* 22.



Uſe. 'The three preceding propoſitions 'are of great uſe, particularly in *Surveying*, 'where 'tis often requir'd to cut from any 'Field, or meaſure any parcel of Acres.

PROB. XXIII.

Upon a given Right Line a b *to deſcribe the Segment of a Circle* a k e b, *which ſhall contain any given Angle, ſuppoſe* 40 *deg.*

Through the point *a* draw the Right Line *a d*, making an Angle of 40 *deg.* with the Line *a b*: From *a* draw *a e* perpendicular to *a d*, and join *e b*; At the point *b* make the Angle *a b f* = *b a f*; from the point *f* as a Center with the diſtance *f a*, deſcribe a Circle which ſhall alſo paſs through *b*, becauſe the Angle *f a b* = *f b a*, then I ſay that the Segment *a e b* is the Segment ſought. For draw from the points *a*, *b*, two Right Lines

Lines *b k*, *a k* to form an Angle in any part of the Periphery as at *k*, the Angle *a k b* = *b a d*, which is equal to 40 *deg.* by Conſtruction.



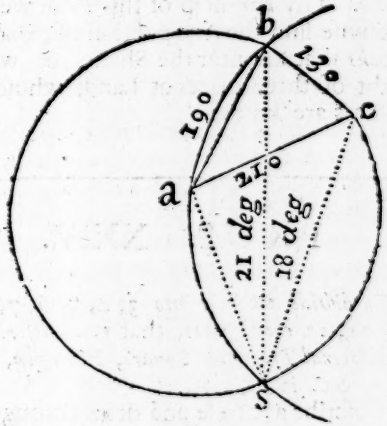
Demp. The Line *a e* is a Diameter of the Circle by (*Theo.* 14.) and therefore the Angle *e b a* is right, and conſequently ∠ *e* + *e a b* = one right. But ∠ *e a b* + *b a d* = one right (*Theo.* 1.) therefore ∠ *e* = *b a d*, but ∠ *e* = *b k a* becauſe they ſtand on the ſame Arch; therefore *a k e b* is the Segment required.

Uſe. 'Upon this *Problem* is grounded 'the Solution of a very uſeful Propoſition, 'Inſerted in the *Philoſophical Tranſactions*, 'Num. 69. *page* 2087, and that after a 'more eaſie and univerſal Method than is 'there expreſs'd. It is as follows.

The

The distance between any 3 Objects being known, and the Angle they make with each other from some known Point or Station in the same Plain, to find the distance of either of those Objects from the said Station.

Let the three Objects be *a*, *b*, *c*, and their Distances and Angles as in the annexed Diagram; upon the Line *a b* describe by the preceding *Prob.* the Segment (*a s b*) of a Circle that shall contain an Angle of 21 *deg.*



also upon the Right Line *b c* describe the Segment of a Circle that shall contain an Angle of 18 *deg.* Now since I must stand in the Periphery of both these Circles to behold

behold the Objects under those Angles, 'tis evident it must be in the point *s* of their Intersection: For if from the point *s* you draw the Right Lines *s a*, *s b*, *s c*, they will form the given Angles, as is manifest from the *preceding Prob.* And therefore if by the Scale, with which you plotted the Triangle *a b c*, you measure the Lines *s a*, *s b*, *s c*, you will get the distance of the said point from either of the Objects, which was the thing required.

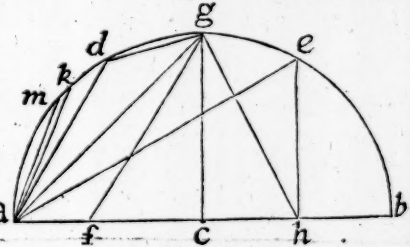
Use. 'By the help of this *Prob.* we may 'likewise find the true position of *Sands* and '*Rocks* that are near the Shoar, or within 'sight of three places at Land, whose Di-'stances are known.

PROB. XXIV.

To divide a Circle into 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 *equal parts*, that is *to describe an Equilateral Triangle, Square, Pentagon, Hexagon*, &c. *in a Circle.*

Describe a Circle and draw through the Center the Diameter *a c b*, then take the distance *c a* and set on the Circumference from *a* to *d*, and from thence again to *e*, and draw *a d*, *a e* the first for the side of an *Equilateral Triangle*, the second for the side of

of a *Hexagon*; croſs *a b* at right Angles with the Line *c g*, and draw *a g* for the ſide of the *Square*; Biſect *c b* in *h*, and from *g* draw *g h*, make *h f* = *h g*, and draw *f g* for the ſide of a *Pentagon* inſcribed in this



Circle; join the points *h e* it gives the ſide of a *Heptagon*; next biſect the Arch *a g* in *k*, and draw *a k* for the ſide of the *Octagon*, this done, divide the Arch *a d e* into three equal parts at *m*, and draw *a m* for the ſide of an *Eneagon* or nine ſided Figure; the Line *f c* is the ſide of a *Decagon*, for the ſide of *Endecagon* or eleven ſided Figure; you muſt have recourſe to Mechanick Artifices, *that is*, to divide the whole Circle into Eleven equal parts, one of which will be the ſide required. Laſtly, join the points *d e* it give the ſides of the *Dodecagon* or 12 ſided Figure.

Many

Many other *Probs.* there are in *Practical Geometry*, which might ſometimes be required, but I omit them, having only inſerted thoſe that are moſt frequent and uſeful, and generally required in the Delineating of any *Geometrick Scheme* or *Figure*.

EUCLID's

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Second BOOK

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EUCLID's

Second BOOK

Geometrically Constructed and Algebraically Demonstrated.

PROP. I.

IF there be two Right Lines *a* and *e*, and one of them as *a* divided into any parts, *suppose b, c, d.* The Rectangle made of the two whole Lines *a* and *e* shall be equal to



the several Rectangles contained under the whole Line *e*, and the Segments or parts *b*, *c*, *d*.

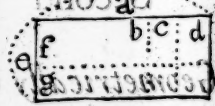
That is, $a e = b e + c e + d e$

Solution Algebraically. Let $a = b + c + d$

Multiply each by e

it produces $a e = b e + c e + d e$

Schol. Hence if there be two right lines *a*, *e*, and both of them cut into several parts, as *a* into the parts *b*, *c*, *d* and *e* into *f*, *g*. The Rectangle under the two whole right Lines, shall be equal to the several Rectangles under all the parts.



That is, $a e = f b + f c + f d + g b + g c + g d.$

Sol. Alg. Let $a = b + c + d$

and $e = f + g$ *by which Mult*

it produces $a e = f b + f c + f d + g b + g c + g d.$



PROP.

PROP. II.



If a Right Line *a* be cut any how into two parts as *b* and *c*, the Rectangles comprehended under the whole Line *a*, and each Segment *b* and *c* are equal to the Square made of the whole Line *a*.

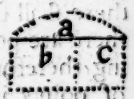
That is, $a\,a = a\,b + a\,c$

Sol. Alg. Let $a = b + c$

Mult. each by a

it produces $a\,a = b\,a + c\,a$

PROP. III.



If a right Line *a* be any way divided into two parts as *b*, *c*. The Rectangle comprehended under the whole Line *a*, and one Segment as *c*, shall be equal to the square of the said Segment, and a Rectangle of both Segments.

That is, $ac = bc + cc$

Sol. Alg. Let $a = b + c$

Mult. each by ——— c

it produces $ac = bc + cc$

Use. 'This with the two foregoing Pro
'positions serve to demonstrate the ordinary
'or common Rule of
'*Multiplication*; Thus,
Let *A* be one given line
'and the Line noted by
'*E* another. If then,
'(as in *Theo.* 7. has been hinted) the Line *E* be
'placed perpendicular to the Line *A*, and in
'that posture be moved parallel to it self
'through every *Physical point* in the Line *A*
'it will produce the Rectangle *AE*; and
'that this motion representeth Multiplica-
'tion, doth thus appear.

'Let there be assumed in the Line *A* any
'number of points Mathematical suppose 8,
'and in the Line *E* let there be taken 3;
'Now 'tis evident that in the Rectangle *AE*
'there shall be as many Lines equal to *E* as
'there are points in the Line *A*: Multiply-
'ing therefore 8 by 3 (*that is* the number
'of points in the Line *A*, by those in the
'Line *E*) it will produce 24 for the num-
'ber of *Mathematick points* in the whole
'*Parallelogram AE*; And this is the reason
'of that common *Notion* in Multiplication,

which

'which is that the *Multiplicand* must be put 'so often into it self as there are Units in the '*Multiplier*, for the Line *A* is increased in 'breadth so often as there were points taken 'in the Line *E*.

'*But here Note*, That by a Mathematical 'point must be understood some part or 'quantity of a Line, and not such a point 'as hath no part. For Example, If I sup-'pose the Line *A* to contain 8 yards, 8 feet 'or 8 inches, then 1 yard, 1 foot or 1 inch 'I term a Point, without considering that 'the same is compoſed of parts; For in 'measuring any line, I must make use of 'some line whose length is known, so like-'wise in the measuring of Superficies and 'Solids, use must be made of a Superficies 'and Solid whose Dimensions are known.

'This being premised, suppose the num-'ber 43 were to be multiplied by the num-'ber 6, having divided 43 into two parts, '*viz.* 40 and 3, I multiply 40 by 6 it produ-'ceth 240, also I multiply 3 by 6 it pro-'duceth 18, which two Rectangles 240 'and 18 are equal to the whole Rectangle 'of 43 by 6, *that is* to 258: The reason of 'which equality is grounded on this *Axiom*, '*That if equal things be multiplied by the 'same or equal things, the products shall be 'equal.*

For $43 = 40 + 3$
Mult. each by ——— 6
it produces $258 = 240 + 18$

PROP. IV.



If a right line a be any how cut into two parts, as b and c, then shall the square of the whole line a be equal to both the squares of the Segments b and c, and to a double Rectangle of the said Segments b and c.

That is, $a\,a = b\,b + 2\,b\,c + c\,c$

Sol. Alg. Let $a = b + c$
Mult. by ——— $a = b + c$
it produces $a\,a = b\,b + 2\,b\,c + c\,c$

Cor. Hence it follows that the square of half any line is equal to the $\frac{1}{4}$ of the Square of the whole line; For the square of $\frac{1}{2}\,a$ is $\frac{1}{4}\,a\,a$.

Use. 'Upon this Propoſition depends the 'method of Operation in the Extraction of 'the Square Root.

PROP.

PROP. VI.

If a ſtraight line a be cut into two equal parts each $\frac{1}{2}a$, and into unequal parts, *viz.* $\frac{1}{2}a + b$ and $\frac{1}{2}a - b$, then is the Rectangle of the unequal parts with the ſquare of the difference of the equal and unequal parts; equal to the ſquare of half the line a.



That is $\frac{aa}{4} = \frac{a}{2} + b \times \frac{a}{2} - b + bb$

Sol. Alg. Let. $\frac{a+b}{2}$ *be the greater part*

Then. $\frac{a-b}{2}$ *is the leſſer part.*

the Product of which is $\frac{aa}{4} + \frac{ab}{2} - \frac{ab}{2} - bb$

add to this bb *it makes* $\frac{aa}{4} + \frac{ab}{2} - \frac{ab}{2} - bb$

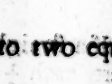
$+ bb$ *which is evidently equal but to* $\frac{aa}{4}$ *when the deſtroying Terms are thrown away.*

Uſe. This with the preceding Propoſition is uſed in demonſtrating the method of finding the Root of an adfected Equation.

PROP.

PROP. VI.

If a straight line a be cut into two eq
parts each $\frac{a}{2}$ and another
right line b be put to the
same directly in one strait
line, then is the Rectangle
of the whole and the line
added taken as one line, and the line add
together with the Square of $\frac{a}{2}$ equal to
square of $\frac{a}{2} + b$ taken as one line.



That is $\overline{a + b} \times b + \frac{a\,a}{4} = \overline{\frac{a}{2} + b} \times \overline{\frac{a}{2}}$

Sol. Alg. $a + b$
Mult. by b

it produces $\overline{a\,b + b\,b}$ *add to this* $\frac{a\,a}{4}$

it makes $a\,b + b\,b + \frac{a\,a}{4}$ *which*

equal to the square of $\frac{a}{2} + b$

For $\frac{a}{2} + b$

Mult. by $\frac{a}{2} + b$

makes $\frac{a\,a}{4} + a\,b + b\,b$

PROP. VII.

If a ſtraight line a be any wiſe cut into two parts, as b and $a-b$, then the ſquare of the whole line added to the ſquare of one of the parts, is equal to two Rectangles under the whole line a, and the ſaid part b, with the ſquare of the other part or Segment $a-b$.



That is, $a\,a + b\,b = 2\,a\,b + \overline{a-b} \times \overline{a-b}$.

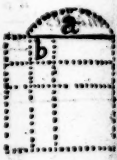
Sol. Alg. $a-b$

Mult. by $a-b$

it produces $a\,a - 2\,a\,b + b\,b$ *add to this* $2\,a\,b$

it makes $a\,a - 2\,a\,b + b\,b + 2\,a\,b = a\,a + b\,b$.

PROP. VIII.



If a ſtraight line a be cut any how into two parts as b and $a - b$, then four Rectangles of the whole line and one of theſe parts with the ſquare of the other part, ſhall be equal to the ſquare of the whole line and firſt Segment taken one line.

That is, $4ab + \overline{a-b} \times \overline{a-b} = \overline{a+b} \times \overline{a+}$

Sol. Alg. $a - b$

Mult. by $a - b$

it produces $aa - 2ab + bb$ *add to this* $4a$

it makes $aa - 2ab + bb + 4ab$ *which*

equal to the ſquare of $a + b$.

For $a + b$

Mult. by $a + b$

makes $aa + 2ab + bb$ *equal to the form*

PROP. IX.



If a ſtraight line a be cut into two equal parts, each half a, and into unequal parts as b and $a - b$, then are the ſquares of the unequal parts b and $a - b$ double to the Summ of the ſquare of half a, and the ſquare of $\frac{a}{a} - b$ the difference.

That is, $bb + \frac{a-b \times a-b}{2} = \frac{aa}{4} + \frac{a}{2} - b \times \frac{a}{2} - b.$

Sol. Alg. $a - b$

Mult. by $a - b$

it produceth $aa - 2ab + bb$ *add to this* bb.

it makes $aa - 2ab + 2bb$

Again

Again $\frac{a}{2} - b$

Mult. by $\frac{a}{2} - b$

it produceth $\frac{a\,a}{4} - \frac{a\,b}{2} - \frac{a\,b}{2} + b\,b$ *add to this*

it makes $\frac{a\,a}{2} - a\,b + b\,b$ *which is but*

the former.

Use. 'This with the foregoing Propo-
'tions are principally used in *Algebra*,
'finding the Root of a Square equal
'Number encreased more by a certain nu-
'ber of Roots.

PROP. X.

If a straight line a be cut into two equal parts, each half a, and another right line b be added to the same directly in one strait line, then is the square of the whole line and the line added taken as one line, dou to the Summ of the squares of half a, a $\frac{1}{2}a + b$ taken as one line.



That is, $\frac{a+b \times a+b}{2} + bb = \frac{1}{2}a+b \times \frac{1}{2}a+b+\frac{1}{4}aa$

Sol. Alg. $a+b$

Mult. by $a+b$

it produces $aa+2ab+bb$ *add to this* bb

it makes $2a+2ab+2bb$.

Again $\frac{a}{2}+b$

Mult. by $\frac{a}{2}+b$

it produces $\frac{aa}{4}+\frac{ab}{2}+\frac{ab}{2}+bb$ *add to this* $\frac{aa}{4}$

it makes $\frac{2aa}{2}+ab+bb$ *which is but half the former.*

PROP. XI.

To cut a line as *b* into *extream* and *mean* Proportion, *that is*, to cut a line into two such parts, that the square of the greater shall be equal to a Rectangle made of the whole line, and lesser parts.

That

That is $\dot{a}\,\dot{a} = b - a \times b$.

Sol. Alg. *Take the thing for done, and the greater part be noted by* a, *then will the lesser be* b — a, *and the Product of the great and lesser* b b — b a *which according to the Proposition must be equal to* a a, that is b b — b = a a, *but by transposition* a a + b a = b *then by adding to either part of the Equation the square of half* b *the co-efficient, to complear the square it will make* a a + b a + $\frac{1}{4}$ b = b b + $\frac{1}{4}$ b b, *and then extracting the square Root of each, we have* a + $\frac{1}{2}$ b = $\sqrt{\ }$: b b + $\frac{1}{4}$ b b *or* a = $\sqrt{\ b\,b + \frac{1}{4}\,b\,b\ }$ — $\frac{1}{2}$ b.

Geometrically, Let the given line be $A = b$, from B erect $B\,E = \frac{1}{2}\,A\,B$ perpendicular to it, and let be drawn $A\,E$; Then (*Theo.* 9.) is $A\,E = \sqrt{\ } : b\,b + \frac{1}{4}\,b\,b$; on the Center E with the distance $E\,B = \frac{1}{2}\,b$ describe the Arch $B\,D$ this cuts from the line $A\,E$ the part $E\,D = \frac{1}{2}\,b$, and then the remaining part $A\,D$ is equal to $\sqrt{\ b\,b + \frac{1}{4}\,b\,b\ } - \frac{1}{2}\,b = a$. And therefore $A\,C$ (= $A\,D$) is the part or Segment sought.



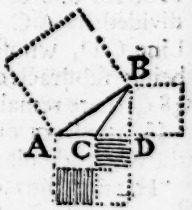
Use. 'This Proposition is requisite for 'the Inscription of some of the Regular Bo- 'dies in a Sphere. It is also necessary to in- 'scribe a *Pentagon* in a Circle, as also a *Pen-*

'*decagon*,

'decagon, and many other neceſſary Practi-
'ces it helpeth to perform. *Father Lucas* of
'St. *Sepulchres* compoſed a whole Book of a
'line cut in this Manner.

PROP. XII.

In an obtuſe angled Triangle *A C B*, the ſquare of the ſide *A B* ſubtending the obtuſe Angle, is greater than the ſum of the ſquares of the ſides *C A*, *C B* including the ſaid Angle, by a double Rectangle made of the ſide *C A*, and that part *C D* betwixt the ſaid obtuſe Angle and the point *D*, where the perpendicular *B D* falleth.



That is, $AB_q = AC_q + CB_q + 2\,ACD$.

Sol. Alg. By *Theo.* 9. $AB_q + AC_q + 2\,ACD + CD_q$, and by the ſame $CB_q = CD_q + BD_q$, therefore (*ax.* 2.) $AB_q = BD_q + CB_q + 2\,ACD$. *W. W. D.*

Uſe. 'By this Propoſition the Perpend
'cular, and conſequently the Area of an
'Triangle may be found by having its thr
'ſides.

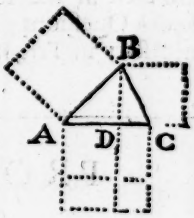
'As ſuppoſe A B be 40 foot, A C 26 foo
'and B C 22 foot, then I ſay the ſquare
'A B will be 1600, the ſquare of A C 67
'B C 484, the Summ of the two latte
'Squares 1160, which being taken fro
'1600 Leaves. 440 for the two Rectangl
'under A C, C D, the one half of which
'220, for one of theſe Rectangles, whic
'divided by A C 26, will give 8 $\frac{6}{13}$ for th
'Line C D, whoſe Square is 71 $\frac{10}{169}$, whic
'being ſubſtracted from the ſquare of B
'484, there remains the ſquare of B D 41
'$\frac{68}{169}$, the Root of which is 20. 3 *prope*, fo
'the Perpendicular B D.

'Having thus obtain'd the Perpendicu
'lar, 'tis eaſie to find the Area of the Tri
'angle, 20. 3 the Perpendicular; For multi
'plied by 13 which is half the Baſe, wil
'give 131. 9 the ſuperficial Content of th
'Triangle.

PROP. XIII.

In any plain Triangle as A B C the ſquare
of the ſide B C ſubtending any acute An-
gle,

gle as *A*, in leſs than the Sum of the ſquares of the other two ſides A B, A C by a double Rectangle made of the ſide C A, and the Segment D A lying betwixt the ſaid acute Angle and the point D where the perpendicular falleth.



That is, BCq = BAq + CAq — 2CAD.

Sol. Alg. By *Theo.* 9. BAq — DAq = BCq (— DCq) — CAq + 2CAD — ADq (*Prop.* 7.) *as is very evident by changing the ſignes of* — DC *and its equal* — CAq + 2CAD + ADq *into their contrarys. And therefore* BAq = BCq — CAq + 2CAD. *So that* (*ax.* 2.) BAq + CAq = BCq + 2CAD *or* BCq = BAq + CAq — 2CAD. *W. W. D.*

Obſerve that the Letters are ſo order'd as to agree with the Figure of *Prop.* 12. in caſe there be an obtuſe Angle in the Triangle.

Note alſo in both caſes, that the double Rectangle of DAC added to the ſquare of BC, or its equal BDq + DCq make the two ſquares of the Lines AB, AC.

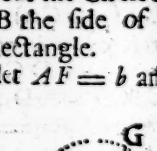
Uſe. 'This, as alſo the preceding *Pro*
'are uſed in the ſolution of ſeveral *Alg*
"*braick* Queſtions; they are likewiſe ver
"neceſſary in Trigonometry.

PROP. XIV.

To find the ſide of a ſquare, equal to a g
ven Rectangle A B C D.

Geo. To F produce A B, making B F =
B D, and biſect A F in E, on E with th
Radius A E or E F deſcribe the Semicirc
A G F, produce D F till it cut the Circle
G, then (*Theo.* 29.) is G B the ſide of
Square equal to the given Rectangle.

Sol. Alg. Draw E G and let $A F = b$ an
$E B = a$, then is $A E = E F = G F = \frac{1}{2} b$;
Alſo $A B = \frac{1}{2} b + a$, and
$B D = B F = \frac{1}{2} b - a$,
and the Rectangle $A D$,
that is $A B D$ equal to
$\frac{1}{2} b + a \times \frac{1}{2} b - a$, *that is*
$\frac{1}{4} b b - a a$ (*Theo.* 9.) For the ſquare Roo
of $\frac{1}{4} b b$ is $\frac{1}{2} b = E G$, and the ſquare Roo
of $a a$ is $a = E B$, therefore $G B$ is th
ſide of a ſquare equal to the Rectangle $A D$
W. W. D.



THE

THE
Doctrine of *Proportion*,

In its various Changes and Combinations Algebraically *Demonſtrated.*

IF Four Magnitudes are proportional, they ſhall alſo be proportional in *Alternation*, *Inverſion*, *Compoſition*, *Diviſion*, *Converſion* and *Mixtion*.

For if $A : a :: B : b$.

Then by
- Alternation $A : B :: a : b$
- Inverſion $a : A :: b : B$
- Compoſition $A + a : a :: B + b : b$ or $A + B : B :: a + b : b$
- Diviſion $A - a : a :: B - b : b$ or $A - B : B :: a - b : b$
- Converſion $A : A \pm a :: B : B \pm b$
- Mixtion $A + a : A - a :: B + b : B - b$, *or as* $A + B : A - B :: a + b : a - b$.

PROP. I.

If 4 Quantities are proportional, the P
duct or Rectangle of the first and fourth,
equal to the Product of the second and third.

Suppose $a : b :: c : d$ then I say that a
$= b \times c$, *that is* $a\,d = b\,c$.

Dem. For (*def.* 48.) $\frac{a}{b} = \frac{c}{d}$ multiply
both by b, it will make $a = \frac{bc}{d}$, and aga
multiply both by d, it will make $a\,d = b$
W. W. D.

For equal things, multiplied by the same
equal things, gives equal Products, up
which *Ax.* the Demonstration both of th
and the following Propositions about Pr
portion usually depend.

In like manner if there be 3 Quantit
in continual proportion, the square of t
mean is prov'd equal to the Product of t
Extreams.

This Proposition shows the reason of th
operation in the Common Rule of Thre
where we multiply the second Term by th
third, or third by the second, and divi
the Product by the first to get the fourth
For if the second multiplied by the thir

be equal to the first multiplied by the fourth, as is here prov'd, then 'tis manifest that the Product of the second and third divided by the first shall give the fourth proportional, for it gives a number, which if the Divisor Multiplies gives the Dividend.

PROP. II.

Numbers or Quantities that have the same Proportion to a third, have also the same one to another.

Suppose $a : b :: c : d :: e : f$, then I say $a : b :: e : f$.

Dem. For $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ (*def.* 48.) & consequently $\frac{a}{b} = \frac{e}{f}$ and therefore $a : b :: e : f$ *W. W. D.*

PROP. III.

If four Quantities are proportional, they shall be so in Alternation.

Suppose $A : a :: B : b$, then I say it will hold Alternately, as $A : B :: a : b$.

 Dem.

Dem. For $A\,b = a\,B$ (*Prop.* 1.) divide both by $b\,B$ the Quote is $\frac{A\,b}{b\,B} = \frac{a\,B}{b\,B}$ or $\frac{A}{B} = \frac{a}{b}$ and therefore $A : B :: a : b$. *W. W. D.*

PROP. IV.

If four Quantities are proportional, they shall be so by Inversion.

Suppose $A : a :: B : b$ then inversly it will hold as $a : A :: b : B$.

Dem. For $A\,b = a\,B$, divide each by $A\,B$ the Quote is $\frac{A\,b}{A\,B} = \frac{a\,B}{A\,B}$ or $\frac{b}{B} = \frac{a}{A}$ and therefore as $a : A :: b : B$. *W. W. D.*

PROP. V.

If four Quantities are proportional, they shall also be so by Composition and Division.

Snppose $A : a :: B : b$, then by Composition $A + a : a :: B + b : b$, and by Division $A - a : a :: B - b : b$.

Dem.

Dem. For $\frac{A}{a} = \frac{B}{b}$ and then $\frac{A}{a} \pm 1 = \frac{B}{b} \pm 1$ *that is* (reducing those mixt Quantities into Fractions) $\frac{A \pm a}{a} = \frac{B \pm b}{b}$ therefore $A \pm a : a :: B \pm b : b$, Which proves both *Compoſition* and *Diviſion*, the *Alternations* and *Inverſions* of which follow from what was before proved.

PROP. VI.

If four Quantities are proportional, they ſhall be ſo by Converſion.

Suppoſe $A : a :: B : b$, then by converſe proportion it will hold as $A : A \pm a :: B : B \pm b$.

Dem. For $\frac{a}{A} = \frac{b}{B}$ (*Prop.* 4.) therefore $1 \pm \frac{a}{A} = 1 \pm \frac{b}{B}$ which by Reduction makes $\frac{A \pm a}{A} = \frac{B \pm b}{B}$ and therefore as $A \pm a : A :: B \pm b : B$, but invertly will hold as $A : A \pm a :: B : B \pm b$, which proves Converſion, the Alternation of which is prov'd (*Prop.* 3.)

PROP.

PROP. VII.

If four Quantities are proportional, they ſh
alſo be ſo by Mixtion.

Suppoſe $A : a :: B : b$, then mixtly 'tw
hold as $A + a : A - a :: B + b : B - b$.

Dem. For by *Converſion* $A : A \pm a :: B : B \pm b$, but by the *Inverſe* of Diviſion $A - a :: b : B - b$. Now becauſe $\frac{A}{A-a} = \frac{B}{B-}$ and $\frac{a}{A-a} = \frac{b}{B-b}$ therefore (*ax.* 1.) $\frac{A}{A-a}$ $\frac{a}{A-a} = \frac{B}{B-b} + \frac{b}{b-b}$ or $\frac{A+a}{A-a} = \frac{B+b}{B-b}$. A
therefore $A + a : A - a :: B + b : B -$
W. W. D.

After the like method may other Au
mentations in Proportion, be prov'd by A
ternating, Inverting and Combining
theſe.

PROP. VIII.

If like Proportionals be multiplied by like Proportional, the Products ſhall be proportional.

Suppoſe $A : a :: B : b$, alſo $E : e :: F : f$, then I ſay 'twill hold as $AE : ae :: BF : bf$.

Dem. For $\frac{A}{a} = \frac{B}{b}$ alſo $\frac{E}{e} = \frac{F}{f}$ then $\frac{EA}{ae} = \frac{BF}{bf}$ and therefore as $EA : ae :: BF : bf$. *W. W. D.*

PROP. IX.

If there be two ranks of Quantities, ſuppoſe A : B : C : D *and* a : b : c : d. *Then if* A : B :: a : b, *alſo* B : C :: b : c. *Laſtly,* C : D :: c : d, *then I ſay it will alſo hold as* A : D :: a : d.

Dem.

Dem. For because $\frac{A}{B}=\frac{a}{b}$ and $\frac{B}{C}=\frac{b}{c}$ also $\frac{C}{D}=\frac{c}{d}$ therefore $\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}=\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}$ For the *Factors* in each being equal, the Products are equal, Expung therefore like Terms in both Dividend and Divisor, and there will remain $\frac{A}{D}=\frac{a}{d}$ and therefore $A : D :: a : d$ which proves proportion of equality.

In like manner may Inordinate proportion be prov'd.

PROP. X.

If there be two ranks of Quantities, as in the Margin then I say it will hold as A : a :: d : D.

A : B :: C : D
a : B :: C : d

Dem.

Dem. For $AD = BC = ad$ (*Prop.* 1.) therefore $AD = ad$; divide both by aD, the Quote is $\frac{AD}{aD} = \frac{ad}{aD}$ or $\frac{A}{a} = \frac{d}{D}$ And therefore as $A : a :: d : D$, which proves Reciprocal proportion.

PROP. XI.

If never so many Quantities are proportional, suppose A : a :: B : b :: C : c. *Then as one Antecedent is to its Consequent, so is the summ of all the Antecedents to the summ of all the Consequents.*

Dem. For if $A : a :: B : b :: C : c$, then $Ab = aB$ and $Ac = aC$: But $Ab + Ac = aB + aC$ *that is,* $b + c \times A = B + C \times a$; which turn'd into a proportion (*p. contra Prop.* 1.) will be as $A : a :: B + C : b + c$, but by Alternation $A : B + C :: a : b + c$, and by Conversion. $A : A + B + C :: a : a + b + c$; and again by Alternation, as $A : a :: A + B + C : a + b + c$. *W. W. D.*

Hence

Hence therefore, if there be a rank of continual proportionals, suppose $A : b : c$, $d : e$, &c. to U, From the preceding Argumentation a *Theorem* may be easily rais'd for finding the Summ of such a Progression.

As let A be the first or least Term, U the last or greatest, and Z the summ of all the Terms, then $Z - U$ the summ of all the Antecedents, and $Z - A$ the summ of all the Consequents; and since by the preceding One Antecedent is to its Consequent, as the summ of all the Antecedents, to the summ of all the Consequents; therefore as $A : b :: Z - U : Z - A$, and therefore (*Prop.* 1.) $AZ - AA = bZ - bU$, *that is* $bU - AA = bZ - AZ$; and dividing both by $b - A$ the Quotes are $\frac{bU - AA}{b - A} = Z$.

Whence arises this General Rule for finding the summ of a Geometrick Progression, divide the Product of the second and last made less by the square of the first; by the second Term made less by the first, the Quote is the summ of the whole Progression.

Hence

Hence 'tis alſo evident, that if the firſt and ſecond Terms are given, the *Ratio* is alſo given; As ſuppoſe *A* the firſt Term, and *B* the ſecond $\frac{A}{B}$ is the *Ratio*, *that is*, the Quantity of the *Ratio*. If only the firſt Term and Quantity of the *Ratio* is given, then the ſecond Term is found by Multiplying the firſt Terms by the *Quantum* of the *Ratio*.

IN

THE MEASURING OF SUPERFICIES AND SOLIDS, *VULGARLY, DECIMALLY* AND *PRACTICALLY*:

WITH

The CUSTOMS used by *Artificers*, for Measuring their several *Works*.

LIKEWISE,

Practical Directions for Measuring *Board* and *Timber*; Making of *VESSELS* to hold any Quantity of *Liquor, Corn*, &c.

LONDON,

Printed in the Year, 1700.

THE MEASURING OF SUPERFICIES AND SOLIDS: *VULGARLY, DECIMALLY* AND *PRACTICALLY*:

Of Measuring.

MEasuring is the Art of finding the *Content* or *Quantum* of any sort of Magnitude, from some given Dimensions.

Now before I give you the Rules for Measuring any sort of Figure, its necessary to inform you that every *Magnitude* is measured by some *Magnitude* that is *Homogenial* or like to it, or that is of the same kind with it; As a Line is measured by a Line, as one, *Lineal*, *Inch*, *Foot*, *Yard*, *Rod*, &c. and a Solid is measured by a Solid, as one Solid or *Cubick*, *Inch*, *Foot*, *Yard*, *Rod*, &c.

So that when you know how many *Lineal Inches*, Feet, &c. are contained in a Line. Or *square Inches*, *Feet*, &c. are contained in a Surface. Or *solid Inches*, *Feet*, &c. are contained in a Solid, then is the *Content* or *Quantum* of either of these Magnitudes said to be known. Other Dimensions besides these there are not in Nature, for Length, Breadth and Depth, include all Space or Capacity.

Hence 'tis evident, that every *Magnitude* must be measur'd by its Homogenial or like Magnitude; *that is*, Longitude by *lineal Inches*, *Feet*, &c. Superficial Measure by *square Inches*, *Feet*, &c. and Solid Measure by *solid Inches*, *Feet*, &c. Now the square and solid Measure is reduced from Measures of Longitude; *that is*, from a Table of Long Measure, may be made a Table both of Superficial and Solid Measure, after the following Method.

A Table of Long Measure.

qu.	I.	Foot.	El. E.	Yard.	El. En.	Rod.
4	12					
48	12	Foot.				
108	27	2 $\frac{1}{4}$	El. E.			
144	36	3	3 $\frac{1}{3}$	Yard.		
180	45	3 $\frac{3}{4}$	1 $\frac{2}{3}$	1 $\frac{1}{4}$	El. En.	
782	198	16 $\frac{1}{2}$	7 $\frac{1}{3}$	5 $\frac{1}{2}$	4 $\frac{2}{5}$	Rod.

If you would know how many ſquare Quarters there are in a ſquare Inch, 'tis but multiplying 4 by 4: If you would know how many ſquare Quarters there is in a ſquare Foot, 'tis but multiplying 48 by 48. Again, If you would know howmany ſolid Quarters is in a ſolid Foot, 'tis but multiplying 4 by 4, and that product again by 4, it gives 64 the ſolid Quarters in one ſolid Inch. After this method may you Calculate a Table of ſquare and ſolid Meaſure.

Now becauſe I ſhall work moſt of the following Examples, both by Vulgar and Decimal Arithmetick, it will here be neceſſary to Incert a Table, ſhewing what Decimal part of a Foot any number of Inches,

Halves and Quarters are to 4 places, *that is* to $\frac{1}{10000}$ part of a Foot.

The Decimal Table for Inches Halves and Quarters.			
Inc.	*Decim.*	*Inc.*	*Decim.*
11	.9167	4	.3333
10	.8333	3	.25
9	.75	2	.1667
8	.6667	1	.0833
7	.5833	$\frac{3}{4}$	.0625
6	.5	$\frac{1}{2}$	.0417
5	.4167	$\frac{1}{4}$	.0208

By this Table you see that 9 Inches is .75 of a Foot, and 6 Inches is .5 of a Foot; If you would have the Decimal of 7 Inches $\frac{1}{2}$ or the like, then you must take the Decimal of 7 Inches, and also of $\frac{1}{2}$ of an Inch and add them both together, so is the total the Decimal of 7 Inches $\frac{1}{2}$.

thus the Decimal of { 7. $\frac{1}{4}$ } { .6041
2. $\frac{1}{2}$ } { .2084
3. $\frac{3}{4}$ } { .3125

But for such as would actually take their Dimensions Decimally, it will be necessary for those to provide themselves with a two foot

foot Rule, on which let each foot be divided into 10 equal parts, and each of those parts into 10 other equal parts, so is the whole Foot divided into 100 equal parts, and is call'd by Artificers the Line of *Foot Measure*, being noted when put on the common two foot Rules, by the Letters F. M. This Rule so divided, is prepar'd to take any Dimensions in Feet and decimal parts of a Foot; and therefore I shall suppose the following Dimensions to be taken by such a Rule that is in Feet and Decimal parts, as well as in Feet and Common Inches.

How to find the Area *of any plain* Superficies, *both by Vulgar and Decimal Arithmetick.*

PROP. I.

To Measure a true *Square*, *long Square*, *Rhombus* or *Rhomboids*.

Rule. *Multiply their length by their true breadth, the Product is the Superficial Content.*

Ex. I. Suppose the side of a true Square be 17 Foot 6 Inch. what's the Content in square Feet.



Vulgarly.

I.

210 *length.*
210 *breadth.*
2100
420
44100 A. I.
F. I.

144) 44100 (306.36 A.
900
36

Decimally.

F. P.
17. 5.
17. 5.
875
1225
175
306.25 A. F. P.

So that the Area or Content of this Square, by the Vulgar way is 306 square Feet, and 36 square Inches; which 36, because $\frac{1}{4}$ of 144, is therefore $\frac{1}{4}$ of a Foot, exactly agreeing

greeing with the Decimal Area, that being 306 Foot, and .25 Inches, which is the ¼ of a Foot also.

Notes.

First, When Feet and Inches are given, they are reduced into Inches by Multiplying by 12, and taking in the odd Inches over and above the Feet; if you would reduce the Inches lower, *that is* into Halves, Quarters or half Quarters, Multiply by 2, 4 or 8. This I thought necessary to premise, because in Questions, I shall give the Dimensions not only in Feet and Inches, but also in Inches, Halves and Quarters, to save the trouble of reducing them, when the Question has such low Dimensions given in it.

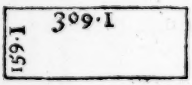
Secondly, Having Multiplied the Length and Breadth together in Inches, it gives the Content in square Inches; if the Length and Breadth were given in Halves or Quarters of Inches, then the Product will be square halves or quarters of Inches: And so whatever Name the Dimensions are when you Multiply them together, of that same Name is the Product, whether it be Feet, Yards, *&c.*

Thirdly,

Thirdly, Becauſe the Area or Content of Dimenſions are generally required in Feet, Yards, or ſome higher Denomination, you muſt divide the ſquare Quarters of Inches by 16, the Quote is ſquare Inches: If in ſquare half Inches you muſt divide by 4 to bring them to ſquare Inches, and if you divide ſquare Inches by 144, the Quote will give ſquare Feet; and if you divide ſquare Feet by 9, the Quote will give ſquare Yards; underſtand the like of reducing it to any other Denomination.

Fourthly, Obſerve that A ſtands for Area, F for Feet, I for Inches, P for parts of a Foot, alſo Q for Quarters of Inches.

Ex. II. Suppoſe the length of a long Square be 25 Foot 9 Inches, or 309 Inches, and the breadth 13 Foot 3 Inches, or 159 Inches, what's the Content in ſquare Inches.



Vulgarly

Vulgarly.

309 *length.*
159 *breadth.*
2781
1545
309
49131 A. I.

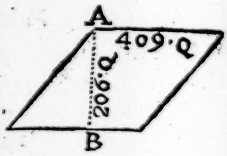
F. I.
144) 49131 (341.27. A.
593
171
27

Decimally.

F. P.
25.75
13:25
12875
5150
7725
2575
341.1875 A. F. P.

Ex.

Ex. III. Suppose the side of a Rhombus be 8 Foot, 6 Inches, $\frac{1}{4}$ or 409 quarters of an Inch, and the true breadth or line A B 4 Foot, 3 Inches $\frac{1}{2}$ or 206 Quarters, what is the Area.



Vulgarly.

Q.
409
206
2454
8180
84254 A. *in square* Q.

16) 84254 (5265 A. *in Sq.* I.
42
105
94
14

F. I. Q.
144) 5265 (36. 81. 14. A.
945
81

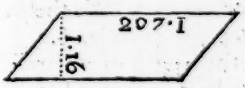
Decimally.

Decimally.

F. P.
8. 521
4. 291
8521
76689
17042
34084
36.563611 A. F. P.

Here (as alſo in ſome others of the following Example) obſerve that the Vulgar and Decimal way do not exactly agree, the reaſon is becauſe the 6 Inches and $\frac{1}{4}$, as alſo the 3 Inches and $\frac{1}{2}$ cannot be truly expreſt in Decimals, and therefore muſt needs make ſome little alteration in the Product.

Ex. IV. Suppoſe the length of a Rhomboids be 17 Foot 3 Inches, or 207 Inches, and the true breadth or Line 8 Foot 7 Inches, or 91 Inches, what's the Content.



Vulgarly.

Vulgarly.

I.

207 *length.*
91 *breadth.*

207

1863

18837 A. I.

F. I.

144) 18837 (130. 177 A.
443
117

Decimally.

F. I.

17. 25
7. 58

13800
8625
12075

130.7550 A. F. P.

The reaſon of Meaſuring any of the preceding Figures, is ſhown in the uſe of Theo. 7.

More Examples.

Suppoſe the ſide of a ſquare be 18 Foot, 11 Inches $\frac{1}{2}$, what's the Area. *Anſw.* 359 *Foot*, 64 *ſquare Inches* $\frac{1}{4}$.

Suppoſe the length of a Parallelogram or long Square be 24 Foot 5 Inches $\frac{1}{4}$, and its breadth 17 Foot 13 Inches, what's the Area. *Anſw.* 437 *Foot*, 120 *ſquare Inches*, 12 *Quarters.*

Sup-

Suppoſe the length of a Rhombus be 35 Foot, 3 Inches $\frac{3}{4}$, and its true breadth 18 Foot, 4 Inches $\frac{1}{2}$, what's its Area. *Anſw.* 648 *Foot*, 124 *ſquare Inches*, 14 *Quarters*.

Suppoſe the ſide of a Rhomboides be 19 Foot, and its breadth 7 Foot, what's the Area. *Anſw.* 133 *Foot*.

PROP. II.

To Meaſure a Triangle.

Rule. 1. *Multiply the whole Baſe by half the Perpendicular; or the whole Perpendicular by half the Baſe, it gives you the Area, Or if you multiply the whole Baſe by the whole Perpendicular, and take half that Product, it gives you the Area.*

Ex. I. Suppoſe the Baſe of a Triangle be 34 Foot 8 Inches, or 416 Inches, and the Perpendicular A B 15 Foot 10 Inches, or 190 Inches, what's the Area.



Vulgarly.

Vulgarly.

I.

207 *length.*
91 *breadth.*

207

1863

18837 A. I.

F. I.

144) 18837 (130.177 A.
443
117

Decimally.

F. I.

17.25
7.58

13800
8625
12075

130.7550 A. F. P.

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Vulgarly.

Vulgarly.

I.

416 *Base.*
95 *half Perp.*
2080
3744
39520 A. I.

F. I.

144) 30520 (274. 64. A.
1072
640
64

Decimally.

F. P.
34. 67
7. 91
3467
31203
24269
274.2397 A. F. P.

If the Perpendicular be not given, it may be found having the three sides, by *Prop.* 12 of the Second Book of *Euclid.*

The reason of measuring a Triangle by any of these Rules, is shown in the use of Theo. 8.

Rule. II

Rule. 2. But in case you would find the Area without the Perpendicular, by having the three sides, do thus; *Add the three sides together and halve the Summ, then subtract each side from that half Sum, and multiply this half Summ by each of the Remainders continually, and the square of this last Product is the Area of the Triangle.*

Ex. II. Suppose the three sides of the Triangle are as in the annexed Figure in the Margin, what's the Area.

35 F.
28 F. } *the sides.*
21 F.

84 F. *the Summ*
42 F. *half Summ*
7 *diff.* 1*st side*
14 *diff.* 2*d side*
21 *diff.* 3*d side*
42 *half Summ*
7
294
14
1176
294
4116
21
4116
8232
86436



) 86436 (294 A. F.
49) 464
584) 2336

PROP. III.

To Measure a Trapezium.

Rule. Divide it into two Triangles by a Diagonal, and drop Perpendiculars thereto, *then multiply the whole Base by half the Summ (or half the Base by the whole Summ) of the Perpendiculars, the Product is the Area.*

Ex. I. Suppose the Dimensions of a Trapezium are as in the Margin, what's the Area.



Vulgarly.

I.

150 } *the Perpendiculars.*
116 }

266 *their Summ*

133 *half Summ*

200 *the Base*

26600 A. I.

P. I.

144) 26600 (184. 104. A.

1220

680

104

Decimally.

Decimally.

F. P.

16.67 *Base*
11.08 *half Sum Perp.*

13336
16670
1667

184.7036 A. F. P.

Note. If two sides of a Trapezium are parallel, *Add them together, and halve the same; this half multiply by the nearest distance betwixt those two sides, the Product gives the Content.* Or if you measure it in the middle betwixt those two parallel sides, it will be the same.

Ex. The Dimensions of a Trapezium whose opposite sides are Parallel, are as in the Margin, what's the Content.

F.
22 } *sides*
18 }
40 *Summ*
20 *half Summ.*
12 *Breadth.*
240 A. F.

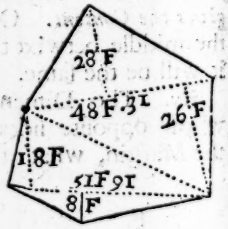


PROP. IV.

To Measure an Irregular Figure.

Rule. This and the like Figures are practically measured by dividing it into Triangles or Trapeziums; which done, *Measure each Triangle severally, and add the several Contents together, so will the Summ be the Contents of the said Figure.*

Exam. Suppose the Dimensions of an Irregular Figure are as in the Margin, what's the Area.



Work according to the preceding Direction, and you shall find 2375 Foot 6 Inches.

The reason of Operation, both in this and the preceding Proposition, is the same of that with Measuring the Triangle, they being all reduced to Triangles.

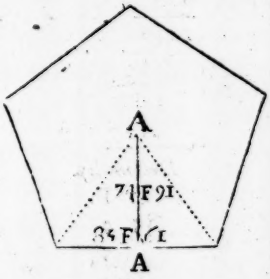
PROP.

PROP. V.

To Measure a Regular Polygon.

Rule. *Multiply the half Summ of all the sides, by the length of a Line drawn from the Center to the middle of any one side, the Product gives the Area.*

Ex. Suppose the side of a Regular Polygon be 85 Foot 6 Inches, or 1026 Inches. The Line A B drawn from the Center to the middle of one side 74 Foot 9 Inches, or 897 Inches, what's the Area.



Vulgarly.

I.

1026 *side.*
5 *number of sides.*
5130 *Summ of the sides.*
2565 *half Summ.*
897 *Line from the Center.*
17955
23085
20520
2300805 A. I.

F. I.

144) 2300805 (15977. 117. A.
860
1408
1120
1125
117

Decimally.

F. P.
85. 5
5
427.5
213.75
74.75
106875
149625
85500
149625
15977.8125 A. F. P.

The

The reason of Measuring any Regular Polygon is very evident, for so many sides as there is in a Polygon, so many Triangles doth it contain, the Bases of which are the Polygons sides; and therefore if we add all the sides (or Bases of the Triangles together) and multiply half the Summ by the Line drawn from the Center to the Circumference, it gives the Area of the Polygon, that is, of all the Triangles at once.

More Examples.

Suppose the side of a Regular Hexagon be 48 Foot, and Line drawn from the Center to the middle of any one side 42 Foot, what's its Area. *Answ.* 6048 *Foot.*

Suppose the side of a Heptagon be 72 Foot, and Line drawn from the Center to the middle of any one side 75 Foot, what's its Area. *Answ.* 18900 *Foot.*

PROP. VI.

To Measure a Circle.

Before I give you the Rule for Measuring this Figure, it will be necessary to shew how, when the Diameter is given, to find the Circumference. *& contra.*

Proportions.

1 : 3. 14159 *or* 113 : 355 :: *Diam. Cir.*
1 : 0. 31831 :: *Circum. Diam.*
1 : 0. 78540 :: *Diam. Square. Area.*
1 : 1. 27324 :: *Area. Diam. ſquar'd.*
1 : 0. 70711 :: *Diam. Side ſqu. inſ.*
1 : 0. 88623 :: *Diam. Side ſqu. equ.*
1 : 0. 07957 :: *Cir. ſqu. Area.*

By theſe Proportions may ſeveral Queſtions relating to the Circle be ſolv'd, and that only by Multiplication, For I have not given thoſe common Proportions of 7 to 22 and the like, which requires both Multiplication and Diviſion, but have put them in ſuch terms, where the firſt is always 1 or unity, by which means I exclude the troubleſom work o Diviſion.

As *Firſt*, Diameter 86 given to find the Circumference, multiply the given Diameter by 3. 14159 and cut five Figures from the Product, it gives the 270. 17674 Circumference.

And here note, that it is at pleaſure to uſe all the Decimals, for 3. 14 will be near enough for any common uſe, but if greater exactneſs be required, you muſt uſe more Decimal Places.

Secondly,

Secondly, Circumference 156 given to find the Diameter, multiply the given Circumference by .31831 it gives the 49.65636 Diameter.

Thirdly, Diameter 24 given to find the Area, ſquare the given Diameter, that is multiply it by it ſelf, and that Product by .7854 it gives (452.1904) the Circles Area.

Fourthly, Area (600) of a Circle given to find its Diameter. multiply the Area by 1.27234 and extract the ſquare Root of the Product, it gives (27.63) the Diameter.

Fifthly, Diameter given to find the ſide of the ſquare inſcribed, Multiply the given Diameter by 70711, the Product is the ſide of the ſquare inſcrib'd.

Sixthly, Diameter given to find the ſide of the Square equal. Multiply the given Diameter by .88623, the Product is the ſide of the Square.

Seventhly, Circumference given to find the Area, ſquare the Circumference, and multiply the Product by .07957 it gives the Area.

Note. *Theſe Tabular Numbers are raiſed and Calculated after the ſame manner as the Tables of Sines Tangents*, &c.

Rule. *Multiply half the Circumference by half the Diameter, it gives the Area of the Circle.*

Ex.

Ex. Suppose the Diameter of a Circle be 16 Foot 3 Inches, or 195 Inches, what's the Area.



Vulgarly.

I.
195 *Diam.*
3.14
780
195
585
612.30 *Cir.*
306.15 *half Cir.*
97.5 *half diam.*
153075
214305
275535
29849.625 A. I. P.

F. I.
144) 29849 (207. 41. A.
1049
41

Decimally.

Decimally.

F. P.
16.25
3.14
6500
1625
4875
51.0250 *Cir.*
25.5125 *half Cir.*
8125 *half Diam.*
1275625
510250
255125
2042009
207.3890625 A. F. P.

Here you see we must use Decimals in this Case, even when we do it vulgarly, and therefore a Circle or its parts is easiest and quickest, measured all by Decimals.

More Examples.

The Diameter of a Circle is 54 Foot 2 Inches, what's the Area. *Answ.* 2304 *Foot* 44 *Inches.*

The Circumference of a Circle is 97 *Foot* 8 *Inches*, what's its Area. *Answ.* 761 *Foot* 91 *Inches.*

PROP. VII.

To measure a Sector.

Rule. *Multiply one of its Legs by half the length of the Arch Line, (or half the Leg by the whole Arch;) the Product is the Content.*

Exam. Suppose the Leg of a Sector be 18 Foot 6 Inches, or 222 Inches, and the length of the Arch line 13 Foot 3 Inches, or 159 Inches, what's its Area.



Vulgarly.

I.
222 *Leg*
111 *half Leg*
159 *Arch*
999
555
111
17642 A. I.

F. I.
144) 17649 (122. 81. A.
324
369
81

De-

Decimally.

F. P.

13. 25
9. 25
6625
2650
11925
1225625 A. F. P.

More Examples.

The reason of Measuring both the whole Circle and Sector, is shown in the case of Theo. *the 6th, they being made up of an infinite number of Triangles.*

The Leg of a Sector is 41 Foot 12 Inches, and the Arch 38 Foot 5 Inches, what's its Area. *Answ.* 797 *Foot* 21 *Inches.*

The Leg of a Sector is 205 Foot, and the Arch 23 Foot 9 Inches, what's the Content. *Answ.* 2434 *Foot* 54 *Inches.*

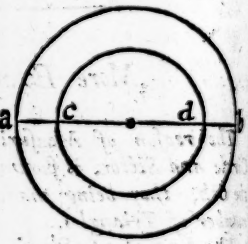
Note. If you would find the Area of a Semicircle, Quadrant, Sectant, *&c.* or any other part of a Semicircle, *'tis but finding the Area of the whole Circle, and then dividing it by* 2, 4, 6, &c. *the Quote gives the Area of the part required.*

PROP.

PROP. VIII.

To measure a Paralled Segment of a Circle.

Rule. *Find the Area of the greater Circle, as also the Area of the lesser, Substract the Area of the greater Circle from that of the lesser, the Remainder is the Area of the Segment sought.*



Ex. Suppose *a b* the Diameter of the greater Circle be 57 foot, and *c d* the Diameter of the lesser Circle 43 foot, what's the Area of the intercepted *Annula* or Ring.

Work, and you shall find .1099 Feet and 49 Inches.

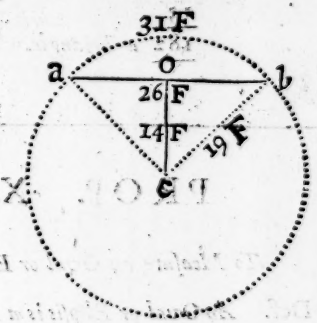
PROP.

PROP. IX.

To measure the Segment of a Circle cut off by a Chord line.

Rule. From the Segment make a Sector, then the reason of measuring the Segment of a Circle is evident; *For find the Area of the Sector, and also the Area of the Triangle, then deduct the Area of the Triangle from the Area of the Sector, the Remainder is the Area of the Segment.*

Ex. Suppose the length of a Segment Arch *a o b* be 31 Foot, and the Chord *a b* 26 Foot, what's the Area.



Foot.

F.

31 *Arch*
15.5 *half Arch*
19 *Leg*

1395
155
2945 *a Sector*
182 *a Triangle*
112 *a Segment.*

26 *Chord* ab
13 *half Chord*
14 *Perpendicular.*

52
13
182 *a Triangle.*

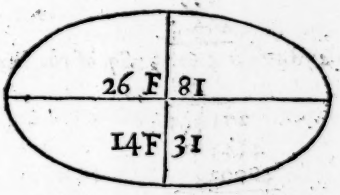
PROP. X.

To Measure an Oval or Elipsis.

Def. *An Oval or Elipsis is a Figure bounded by one Curve Line, generated from the Section of a Cone, by a Plain passing through both sides.*

Rule.

Rule. *Multiply the longeſt Diameter by the ſhorteſt, and Extract the Square Root of the Product, it gives the Diameter of a Circle, whoſe Area is equal to the Area of the Elipſis.*



Ex. Suppoſe the longeſt or tranſverſe Diameter of an Elipſis be 26 Foot 8 Inches, and the ſhorteſt or conjugate 14 Foot 3 Inches, what's the Area or Content.

I.

320 *long Diam.*
171 *ſhort Diam.*
320
2240
320
) 54720 (233 *Diam. Circle equal.*
43) 147
463) 1820
431

I.

233
233
699
699
466

As 1 to 4785 so is 54289 Sq. of the Diam.
785

271445
434312
380023

42616.865 Area of the Circle equal to the Oval in I.

F. I.

144) 41831 (295. 136. A. of the Elipſis
1381
856

More Examples.

Suppoſe the longeſt Diameter of an Elipſis is 140 Foot, and the ſhorteſt Diameter 87 Foot, what's the Area. *Anſw.* 9544 Foot 109 Inches.

Suppoſe the longeſt Diameter of an Elipſis be 86 Foot 2 Inches, and the ſhorteſt 59 Foot 1 Inch, what's the Content. *Anſw.* 3395 Foot 101 Inches.

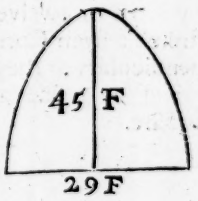
PROP.

PROP. IX.

Def. *A Parabola is a Figure bounded by two Lines, one of which is curve, the other straight; it is generated by cutting a Cone with a plain Parallel to the side thereof, as the Figure.*

To Measure a Parabola.

Rule. *Multiply the line by the line and that Product again by 2, Divide this last Product by 3, the Quote is the Area of the Parabola.*



Ex. Suppose the breadth of the Adjacent Parabola be 29 Foot, and the Height 45 Foot, what's the Area. F.

```
  29
  45
 145
 116
1305          3) 2610 (870 F. A.
   2             21
2610
```

The Demonstration of the reason of Measuring the two last Figures, is founded on the Doctrine of the Conick Sections; where 'tis proved, that a mean Proportional betwixt the longest and shortest Diameter of an Elipsis, is the Diameter of a Circle, whose Area is equal to that of the Elipsis; and that the Parabola is two thirds of its circumscribing Parallelogram.

And here Note, that the best way for practically taking the Dimensions of any of the forementioned Figures, whether upon the Ground, Walls or Ceilings, is either by a ten or twelve Foot Rod, or by a Line stretch't from Corner to Corner; For Perpendiculars, they are actually dropt or raised by a line applied to one side of a Square.

THE

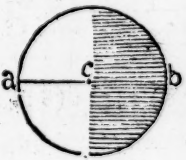
THE MEASURING OF SOLIDS.

PROP. X.

To Measure a Globe or Sphere.

Rule. *First find the Circumference, which multiply by the Diameter, the Product is the Superficial Content; this Superficial Content Multiplied by* $\frac{1}{6}$ *of the Diameter, the Product is the Solid Content.*

Ex. Suppose *a b* the Diameter of a Sphere is 2 Foot 6 Inches, what's the solid Content.



Decimally.

3.141
2.5 *Diam.*
15705
6282
7.8525 *Circum.*
2.5
392625
157050
19.63125 *Superficial Content.*
.417 $\frac{1}{6}$ *of the Diam.*
13741875
1963125
7852500
8.18623125 *Solid Content in* F. & P.

Vulgarly.

3.141
30 *Diam. in* I.
94.230 *Circum* in I.
30
2826.900 A. *of its Surface.*
5 $\frac{5}{6}$ *of the Diam.*
14134.500 *Solid Content of the Sphere in* I.

1728) 14134 (8 F. 310 I. *Content of the Sph.*
310

More

More Examples.

Suppoſe the Diameter of a Sphere is 63 Foot, what's the Solid Content. *Anſwer* 130924 *Foot* 1036 *Inches.*

Suppoſe the Diameter of a Sphere is 86 Foot 6 Inches, what's the Solid Content. *Anſw.* 338881 *Foot* 998 *Inches.*

Suppoſe the Circumference of a Sphere is 8 Foot 6 Inches, what's the Solid Content. *Anſw.* 12 *Foot* 767 *Inches.*

Notes.

The Content of any Solid being found in Inches, Divided by 1728 the ſolid Inches in one Foot, it gives the Content in ſolid Feet, if the Inches that remain is half or a quarter of 1728, then its half or a quarter of a Solid Foot.

Secondly, If the Circumference of a Sphere be given, then find the Diameter, as before has been Taught.

Thirdly. In the vulgar way of Meaſuring this, and other of the following Solids, that in Multipling or Dividing we reject the Decimals, which ariſe by finding the Circumference or Diameter.

The Reason of Multiplying the Circumference by the Diameter, for finding the Superficial Content, is because the curve surface of the Circumscribing Cylender is equal to the curve surface of the Sphere: And the reason of finding the solid Content, is evident from Theo. 44.

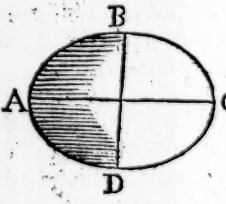
PROP. XI.

To Measure a Spheroid.

Def. *A Spheroid is a solid Oval, generated by the motion of an Elipsis, upon its Transversa or longest* Diameter, *being in shape like a Pumpkin, as the Figure* A B C D. *For Measuring of which this is the*

Rule, By the Circumference and Diameter find the Area of the lesser Circle; this Area Multiply by $\frac{2}{3}$ of the longest Diameter, the Product is the Solid Content.

Exam. Suppose the shortest Diameter of a Spheroid be 6 Foot 7 Inches, and the longest 9 Foot 9 Inches, what's the Content.



Vulgarly

Vulgarly.

3.141
79 I.
28269
21987
248.139 *Circum.*
124.069 *half Circum.*
395 *half Diam.*
620345
1116621
372207
4900.7255 A. I.
78 *two thirds of the long Diam.*
392058040
343050785
382256.890 *Content in* I.

F. I.

1728) 382256 (221. 368 *Solid Content of the Spheroid.*
3665
2096
368

Desi-

Decimally.

6.583
3.141
6583
26332
6583
19749
20677203 *Circum.*
10338601 *half Cir.*
3291
10338601
93047409
20677202
31015803
34024335891
65
170121679455
204146015346
221.1581832915 *Solid Cont. in* F, P.

Another Example.

Suppose the longest Diameter of a Spheroid be 24 Foot, and the shortest Diameter 18 Foot, what's the Solid Content. *Answ.* 4071 F and $\frac{1}{2}$.

The reason of Measuring this Figure depends upon the Knowledge of Doctrine of Conick Sections.

PROP. XII.

To Measure all sorts of Prisms, as Parallelepipedons, Cubes, Cylenders, &c.

Under this Figure (as before intimated) is comprehended the Cube, Parallelepipedon, Cylender, and all other Solids, that are as big at one end as the other; For Measuring which this is the

Rule. *Find the Area of the Base, whether it be Triangle, Square, Circle, or whatever else; this Area multiply by the length or height of the Prism, and the Product gives you the solid Content.*

Ex. 1. Suppose the breadth of a Parallelepipedon L be 3 Foot 9 Inches, its thickness 2 Foot 3 Inches, and its length 18 Foot 6 Inches, what's the Solid Content.



Vulgarly.

Vulgarly.

45 *breadth in* I.
27 *thickneſs in* I.
315
90
1215
222 *length in* I.
2430
2430
2430
1723) 269730 (156 F. 162 I. *Solid Cont.*
.9693
10530
162

Decimally.

275
2.25
1875
750
750
8.4375
18.5
421875
675000
84375
156.09375

The

The Cube is alſo Meaſured by this Rule; and therefore, ſince its length, breadth and thickneſs are equal, 'tis but Cubeing of its ſide; *That is*, Multiplying one ſide three times into it ſelf, the Product is the ſolid Content.

Exam. II. Suppoſe the ſide of a Cube be 7 Foot, what's the ſolid Content.

F.
7
7
49
7
343 F. *Solid Content.*



Exam. III. Suppoſe the Diameter of the Baſe of a Cylender be 6 Foot 9 Inches, and the Perpendicular height 12 Foot 3 Inches, What's the ſolid Content.



Vulgarly.

Vulgarly.

3.141
81 *Dium. in* I.
3141
25128
254.421 *Circum.*
127.210 *half Cir.*
81
127210
1017680
10304.010
5152.005 *Area of the Base.*
157 *height in* I.
36064035
20608020
5152005
757344.735 *Content in* I.

1728) 757344 (438 F. 480 I. *Solid Content of the Cylender.*
.6614
14304
.480

Decimally.

Decimally.

3.142
6.75
15705
21987
18846
21.20175
10.60087
3.38
8480696
3180261
3180261
35.8309406
12.25
1791547030
716618812
716618812
358309406
438.929022350 *Solid Cont. in* F. P.

Exam. IV. Suppose the side of an Equilateral, or a triangular Prism be 2 F. 6 I. and its length or height 23 F. 9 I. What's the solid Content.



Vulgarly.

Vulgarly.

30 } *the 3 sides in* I.
30 }
30
90 *Summ in* I.
45 *half Summ*
15 *Differ.*
225
45
675
15
3375
675
10125
15
50625
10125
)151875 (389 *Root.*
68)618 285 *height* I.
769) 7475 (1945
554 3112
778
110865 *Cont.* I.

1728) 110865 (64 F. 273 I. *Solid Content.*
7185
273

Dect-

Decimally.

2.5 }
2.5 } *The 3 sides.*
2.5 }
7.5 *Summ.*
3.75 *half Summ.*
1.25
1875
750
375
46875
1.25
234375
93750
46875
5.859375
1.25
29296875
11718750
5859375
7.32421875 (2.7063 *Area of Base*
47) 332 23.75
5406) 34218 135315
54123) 178275 189441
15906 81189
54126
64.270625 *Solid Cont.*

More Examples.

Suppose the breadth of a Parallelepipedon is 22 F. 6 I. and the thickness 14 F. 3 I. and the height 32 F. 9 I. what's the solid Content. *Answ.* 10500 F. 810 I.

Suppose the Diameter of a Cylender is 46 F. 7 I. and the Perpendicular height 124 F. 5 I. What's the solid Content. *Ans.* 212005 F. 1079 I.

Suppose the side of a Cube is 16 F. 9 I. what's the solidContent. *Answ.* 4699F. 729I.

Suppose the side of an Equilateral Triangular Prism is 3 F. 10 I. and the length 37 F. what's the Solid Content. *Answer* 235 F. 624 I.

After the same manner is the Content of all Polygonous Prisms found; that is, *such whose Bases contain* 5, 6, 7 *or* 8 *sides.*

PROP. XIII.

To Measure a Cone or Pyramid.

Rule. *First find the Area of the Base, whether it be Triangular, Quadrangular or Circular, which last is the Base of the Cone; then multiply this Area by* $\frac{1}{3}$ *of the Perpendicular height, the Product is the solid Content.*

Ex.

Ex. 1. Suppose the side of a square Pyramid be 2 Foot 6 Inches, and the Perpendicular height 24 Foot 9 Inches, what is the solid Content.



30 } *Side of the Square in* I.
30 }
900 *Area of the Base.*
99 $\frac{1}{3}$ *of the height in* I.
8100
8100
89100 *Solid Content in* I.

1728) 89100 (51 F. 972 I. *Solid Cont.*
2700
972

Decimally.

2.5
2.5
125
50
625
825
3125
1250
5000
51.5625 F. P. *Solid Content.*

Exam. II. Suppose the Diameter of a Cone be 4 Foot 6 Inches, and the Perpendicular height 18 Foot 9 Inches, what's the solid Content.



Vulgarly.

3.1414
54 *Inches Diameter*
125656
157070
169.6356 *Cir.* I.
84.82
27
59374
16964
2290.14 A. *of the base in* I.
77 I. $\frac{1}{2}$ *of the height*
16 03098
1603098 F. I.

1728)171634078(99.688. *Sol. Cont.*
16240
688

Deci-

Decimally.

3.141
45
15705
12564
14.1345
7.0672
2.25
353360
.141344
141344
15.901200
6.25
79506000
31802400
95407200
99.3825 A. F. P.

Note. If you would find the Area of the Curve Surface of a Cone, 'tis done *by multiplying the whole Circumference by half the ſlant height, or half the Circumference by the whole ſlant height, the Product gives the ſolid Content.*

For Example. Suppoſe, as in the preceding Cone, the Circumference be 166 Inches, and the ſlant height 18 Foot 9 Inches, What's the Area of the Curve Surface.

166 *Circumference in* I.
83 *half Circumference.*
225 *ſlant height in* I.
415
166
166
18675 A. *in* I.

F. I.
144) 18675 (129. 99. A.
427
1395
99

More Examples.

Suppoſe the longeſt ſide of the baſe of an oblong Pyramid be 42 Foot 9 Inches, and the ſhorteſt ſide thereof 16 Foot 4 Inches, and the Perpendicular height 63 Foot 3 Inches, what the Solid Content. *Anſwer* 14716 Foot.

Suppoſe the Diameter of the baſe of a Cone be 36 Foot 6 Inches, and the Perpendicular height 48 Foot 9 Inches, what's the ſolid Content. *Anſw.* 17082 Foot.

Sup-

Suppoſe the Circumference of the Baſe of a Cone be 78 Foot, and the ſlant height 96 Foot, What's the Area of the Curve Surface of that Cone. *Anſw.* 3744 Foot.

PROP. XIV.

To Meaſure the Fruſtrum of a Pyramid or Cone.

Def. *The Fruſtrum of a Pyramid or Cone, is only the remaining part when the Top is cut off, ſuch are the Figures* X *and* Y *in the following Examples.*

Rule. *Find the Area of both Baſes, then multiply the one by the other, and extract the ſquare Root of the Product; This ſquare Root add to the Area of both Baſes, the Sum multiply by* ⅓ *of the Perpendicular height, the Product is the Solid Content.*

Exam. 1. Suppoſe the ſide of the greater Baſe of the Fruſtrum of a true Square Pyramid be 3 Foot 6 Inches, or 42 Inches, the ſide of the leſſer Baſe 2 Foot 3 Inches, or 27 Inches, and the Perpendicular height 6 Foot 9 Inches or 81 Inches, what's the Solid Content.

27 } *side lesser base.*
27 }
189
54
729 A. *lesser base.*



Vulgarly.

42 } *side greater.*
42 }
84
168
1764 A. *greater base.*
729 A. *lesser Base.*
15876
3528
12348
1285956 A. *of both bases.*

) 1285956 (1134
21) 28 1764 A. *great base.*
223) 759 729 A. *less base.*
3627
2264) 9056 27 $\frac{3}{8}$ *of the height.*
25389
7254
97929 *Solid Content in* I.

1728) 97929 (56 F. 1161 I. *Solid Cont.*
11529
1161

Deci-

Decimally.

3.5	2.25
3.5	2.25
175	1125
105	450
12.25	450

5.0625
12.25
253125
101250
101250
50625
62015625

62015625 (7.879

148) 1301	12.25
1567) 11756	5.0625
15745) 78725	25.1875
	2.25
	1259375
	503750
	503750
	56.671875 F. P. *Sol. Cont.*

Ex.



Exam. II. Suppose the Diameter of the greater Base of a Cone be 19 Inches, and of the lesser base 14 Inches, and perpendicular height 48 Inches, what's the solid Content.

I.	P.	
283.	52	A. *of the greater base.*
153.	93	A. *of the lesser base*
43642.	7336	*Product of both bases.*
208.	9	*Sq.* √ *of the Product of both bases.*
283.	52	
153.	93	
646.	35	
	16	*a third of the height.*
387810		
64635		
1034160		

1728) 10341 (5 F. 1701 I. *Solid Cont.*
1701

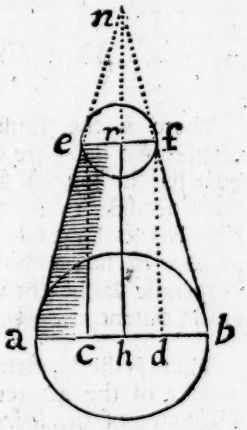
And here note that not only in this, but in several of the preceding and following Examples, we have rejected part or all the decimal places, when we come to multiply or divide them by other Numbers; for which reason several of the Area's are not exact, tho' near enough for common Business. For

For the Reason of the Rule given for Measuring these Fruſtrum. See *Oughtred's Clavis. Page* 141, *&c.*

There is another method for Measuring these kind of Figures, which is done by compleating the Fruſtrum; that is, by adding another perfect Cone or Pyramid to it, and ſo make the Fruſtrum a perfect Cone or Pyramid it ſelf.

For Example, Let the Figure *a b f n e* be the Fruſtrum of a Cone.

Now it is evident, that if by *Prop.* 14 I find the ſolid Content of the whole Cone, and alſo the Content of the added Cone *e n f*, and then ſubſtract the Area of the leſſer Cone from the Area of the greater, the Remainder is the Area of the Fruſtrum *a b f n e*.



The method of compleating the Fruſtrum is thus.

As *a c* the half difference of the Diameters to *c e* the height of the Fruſtrum, ſo is *e r* the half Diameter of the leſſer Baſe to

r n

r n the height of the whole Cone, by *Theo.* 18.

Having thus obtained the height of the whole Cone, deduct the height of the Frustrum therefrom, and the Remainder is the height of the little Cone that must be added.

This Proportion will hold good for square Pyramids, if instead of the half difference of the Diameters, you use the half difference of the sides.

More Examples.

There is the Frustrum of a Pyramid whose two Bases are Oblongs, the greater Base has its longest side 21 Foot 6 Inches, the short side 14 Foot 4 Inches; the lesser Base has its long side 9 Foot 6 Inches, and its shortest side 6 Foot 4 Inches, and the Perpendicular height is 36 Foot, what's the solid Content. *Answ.* 6054 F.

There is the Frustrum of a Cone, the Diameter of the greater Base is 16 Foot, the Diameter of the lesser is 14 Foot, and the Perpendicular height 24 Foot, what's the Solid Content. *Answ.* 4248 F.

PROP.

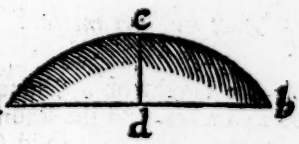
PROP. XV.

To Measure the Segment of a Sphere.

Def. *The Segment of a Sphere or Globe, is a piece cut off like the slice of an Apple, such is the Figure* a b c d.

Rule. *Multiply the treble height of the Segment, which is* c d *by the square of the Radius* a d *or* d b *of the Segments Base, adding to the Product the Cube of the Segments height: The Summ multiplied by .5235 will give the Solid Content of the Segment.*

Exam. What's the solid Content of the Segment, whose height *c d* is 3 Foot, and Diameter *a b* 12 Foot.



67	
65	*Radius.*
36	*Sq. of Radius.*
9	*Ineb. height.*
324	
27	*Cube of the Height*
351	
5235	
1755	
1053	
702	
1755	
183.7485	F. *Solid Content.*

If you find the Area of its Curve Surface then Multiply the Circumference of the Base by the Altitude of the Segment; The *Reason of which is, because the curve Surface of the Circumscribing Cylender is equal to the curve Surface of the Segment.*

Another Example.

Suppose the height of a Segment of a Sphere be 24 Inches, and the Radius of the Base 68 Inches, what's its solid Content? *Answ.* 105 F. 84 I.

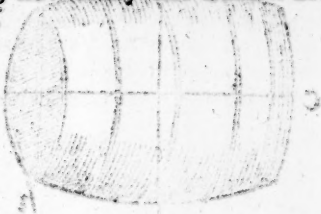
PROP

PROP. XVI.

To Measure the middle Frustrum of a Spheroid, in which form are most of our Common Beer and Ale Casks.

Rule. *Find out the Area's of the two Circles belonging to the Diam. at head & Bung, and then add to* $\frac{1}{3}$ *of the lesser Area* $\frac{2}{3}$ *of the greater Area, the Summ multiply by the length of the Cask, the Product is the Content in Cubick Inches; which if you divide by 282 gives the Content in Ale Gallons, or by 231 give the Content of the said Cask in Wine Gallons,*

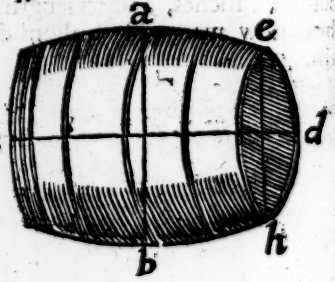
Exam. Suppose *e b* the head Diameter of a Cask be 24 Inches, *a b* the Bung Diameter 34 Inches, and its length *c d* 48 Inches, how many Ale Gallons doth it contain.



Vulgarly.

3.1415
34 *Diam. at Bung.*
125660
94245
106.8110 *Cir.*
53.4055 *half Cir.*
17 *half Diam.*
3738385
534055
907.8935 A. *of the Cir. at Bung.*
605.2623 *two thirds.*

3.1415
24 *Diam at head.*
125660
62830
75.3960 *Cir.*
37.698 *half Cir.*
12 *half Diam.*
451.376 A. *of the Cir. at head.*
159.458 *One third.*



605.6262 *two thirds.*
150.458 *one third.*
756.0842
48 *length.*
60486736
30243368
36292.0416 *Content in Cube Inches.*

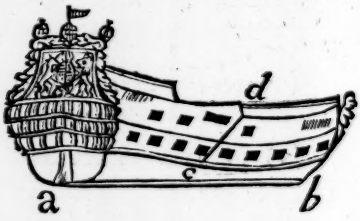
282) 36292 (128 *Ale Gallons.*
809
2452
196

PROP. XVII.

To Measure a Ship, *that is*, to find her Tunnage.

Rule. *Multiply the breadth of the Ship*, that is, *the length of the Midship Beam by the half breadth, and that Product again by the length of the Keel it gives her solidity in Cubick Feet; then divide this last Product by 94, the Quote will give you the Tunnage of the Ship.*

Exam. Suppose a Ships Keel as *a b* is 84 Foot, and the breadth or length of the Beam *c d* 26 Foot, what's her Content.



26 *breadth.*
13 *half breadth.*
78
26
338
84 *length of the Keel.*
1352
2704
94) 28392 (302 *Tuns.*
0192
04

Others have this Rule for Measuring a Ship.

Measure the length of the Keel, Midship Beam and Depth of the Hold, these three numbers Multi-

Multiply one by the other; Divide the Product by 95, *and the Quote will give the true Burthen of any Merchants Ship: For Men of War divide by* 100, *becauſe in them there is an Allowance made for* Ordnance, *Maſts and Sails*, &c.

PROP. XVIII.

To find the Content of any Irregular Solid, whether Lead, Iron, Stone, Wood, &c.

Rule. *To effect this, take a convenient Veſſel, and pour into the ſaid Veſſel ſo much Water as will cover the Body you would Meaſure, then make a mark on the ſide of the Veſſel at the ſurface of the Water; which done put the Body (whoſe ſolid Content is required) into the Water, and it will cauſe it to riſe up above the marke; then take ſo much water out of the Veſſel as is raiſed above the mark, and put the ſame into an hollow Cube or other ſquare Veſſel, which you ought to have in readineſs for ſuch a purpoſe; the ſolid Content of this Water in the Cube, is the ſolid Content of the Body required.*

Some useful Proportions for Measuring of Solids.

As 1 : 0.523599 :: Diam. Cub'd : Solid. Sph.
As 1 : 0.016887 :: Periph. Cub'd : Solid. Sph.
As 1 : 1.9086 :: Solid. Sph. to Cube Diam.
As 1 : 0.523599 :: the square of the conjugate Diameter of a Spheroid, multiplied into the tranverse or long Diameter to the solid Content of the Spheroid.

As 1 : 0.785399 :: the square of the Diameter of the Base of a Cylender, multiplied into its length, to the solidity of the Cylender; Or so is the square of the Diameter of the base of a Cone, multiplied by $\frac{1}{3}$ of the Axis, to the solid Content of the Cone.

These Proportions are fram'd after the same manner as those in *page* 184, and their use to the same purpose; an Instance of which I shall here insert.

First, Diameter 24 of a Sphere given, to find the solid Content. Multiply the Cube of the given Diameter by .523599, it gives, rejecting the Decimals 7238 the solid Content of the Sphere.

Secondly, Circumference of a Sphere 24 given, to find the Solid Content. Multiply the Cube of the given Circumference by .016887, it gives 42 the solid Content.

Thirdly,

Thirdly, The Solidity of a Sphere 126 given to find its Diameter, Multiply the given Solidity 126 by 1.9086, the Product gives 1240.4836 the Cube of the Diameter, the Cube Root of which is 132.25, the Diame- of the Sphere required.

Fourthly, Conjugate Diameter 4.1, and tranverſe Diameter 7.2 of a Spheroid given, to find the ſolid Content. Multiply the ſquare of the Conjugate by the Tranverſe Diameter, and that Product again by .523599, it gives 63 the ſolid Content.

Fifthly, The Baſe of the Diameter of a Cylender is 12, and the height 18 given, to find the ſolid Content. Multiply the ſquare of the Diameter of the Baſe by the Height, and the Product again by .785399, the Product 2035 is the ſolid Content.

Sixthly, If the Diameter of the Baſe and Height of a Cone are as in the preceding Caſe of the Cylender, then the ſolid Content of the Cone is 678 *near*.

DIRECTIONS FOR THE MEASURING Most Sorts of *Artificers WORKS.*

THE Principal Works that are generally Measur'd, are Masons, Carpenters, Bricklayers, Plaisterers, Joiners, Glaziers, Painters and Paviers, several of which give their Contents by different Measures; *that is*, some in Rods, others in Squares, others in Yards and some in Feet, both Square and Running Measure.

By Running Measure is meant length without Breadth; *that is*, so many Foot long is so many Foot Running or Line Measure.

Now when we take the Dimensions of any kind of Work, we do it by Feet; *that is*, we say so many Feet and Inches by so many

many Feet and Inches, and from such Dimensions we cast up the Content in square Feet and Inches, after which we reduce it to square Rods, Yards, &c. according to the Measure the Work is measured by,

The Method used in setting down the Dimensions as we take them, and then casting them up, is as follows.

Dimensions.

F.	I.	P.
24.	10.	0
8.	9.	0
3.	5.	6
2.	8.	9

The meaning of it is this, that 24 Foot 10 Inches is to be Multiplied by 8 Foot 9 Inches; And 3 Foot 5 Inches and 6 Parts, is to be multiplied by 2 Foot 8 In. and 9 Parts.

The Method of casting them up is thus, first multiply 24 Foot by 8 Foot, it gives 192 Foot; then 10 Inches by 8 Foot gives 80 Inches, which divide by 12 it gives 6 Foot 8 Inches; set this under 192 Foot. And here note that the 80 Inches are not square Inches, but such pieces that are an Inch broad and one Foot long, as is very evident from a little consideration: Then multiply the 24 Foot by 9 Inches, it gives 216 Inches, which divided by 12 gives 18

Foot, this you must set under the former 6 Foot 8 Inches: Lastly, Multiply 10 Inches by 9 Inches, the Product is 90 parts, which divided by 12 gives 7 Inches 6 Parts; set this down also under the former in its proper Column. Having thus cast up these Dimensions, and set down the Content right against it, according to the preceding Direction, add up each Row, and set down all above 12, carrying the Twelves to the next Row, just as you do in the Addition of Pence till you come to the last Row, which must be added up all as Integers.

After this manner is the Dimensions of all kinds of work both set down and cast up. See the Example following.

Dimensions.

F.	I.
24.	10.
8.	9.
192:	
6:	8
18:	0
	7:06
217:	03:06

By this you see that the Product of 24 Foot 10 Inches by 8 Foot 9 Inches, is 217 Foot

Foot 3 Inches 6 Parts, this is ſet down againſt its Dimenſions thus F. I. F. I. P.

proceding after this manner with the reſt, ſetting down the proper Products againſt their reſpective Dimenſions; which being done, we caſt up all the Products into one total Summ of Feet, and by Diviſion turn it into ſquare Rods, Yards, *&c.*

24. 10. } 217. 3. 6
8. 9. }

If you take your Dimenſions nearer than Inches, as in ſome ſort of Choice Work (particularly Glazing) you do, that is to Halves and Quarters, then you muſt for a quarter of an Inch ſet down 3 parts, and for half an Inch 6 Parts, and for ¾ of an Inch 9 Parts, for we count 12 parts in an Inch, as well as 12 Inches in a Foot; after which caſt up the Dimenſions, as in the following Example.

F.	I.	P.
3.	5.	6
2.	8.	9
6.	10.	
	1.	
2.	3.	4
		4
	2.	3
		3. 9
		4. 6
9:	05:	03: 01: 06

But

But that you may not miſtake the nature of the Product in multiplying ſeveral Dimenſions together, I ſhall give you this general Rule.

Feet multiplied by Feet gives Feet.
Feet by Inches gives Inches.
Feet by Parts gives Parts.
Inches by Inches gives Parts.
Inches by Parts gives Seconds.
Parts by Parts gives Thirds.

Note, In caſting up your Dimenſions, ſome contractive ways may be uſed; For Inſtance, in the firſt of the preceding Examples, inſtead of multiplying the 24 Feet by 9, and dividing by 12, take but $\frac{3}{4}$ of 24 and it performs the buſineſs: Alſo if you are to multiply any number of Feet by 6 Inches, take half the ſaid Feet, and 'tis the ſame thing as if you had multiplied by 6 and divided by 12. In like manner to multiply any number of Feet by 3 Inches, 'tis but taking $\frac{1}{4}$ of the Feet, and its the ſame thing as if you had multiplied by 3 and divided by 12.

And note, that if in any ſort of Work there be two Dimenſions alike, then is ſuch Dimenſions not ſet twice over, but thus,

4. 3 } twice.
2. 6 }

Having

Having premiſed theſe general Notions, I ſhall now paſs to the particular Methods and Cuſtoms uſed by Artificers, for Meaſuring each of their Works.

Inſtructions for Meaſuring of Brick-Work.

IF a Brick Wall be one Rod ſquare and a Brick and half thick, then is ſuch a piece of Brick work a Rod of Brick work; but if a piece of Wall be a Rod ſquare, and more or leſs then one Brick and ½ thick, then is the ſaid piece of Wall accordingly more or leſs than one Rod of Brick work; And therefore if a Brick Wall be thicker or thinner than one Brick and ½, it muſt be reduced to that thickneſs; For in a piece of Brick work one Rod ſquare and three Bricks thick, there is two Rods of Brick work, by reaſon 'tis double the thickneſs it ſhould be; which work of Reduction makes the meaſuring of Brick work more troubleſom than any other ſort of Work.

And here note, that by 1 B. ½ is meant a Brick in length and another in breadth, that is 9 Inches, and 4 Inches and ½, which with a joint of Mortar makes 14 Inches, the length of one Brick and ½.

This

This Work is generally measur'd by the Rod, *that is,* 'tis commonly brought into square Rod, which contains 272 Foot $\frac{1}{4}$, Or 272 Foot 36 Inches, and is produced by Multiplying 16 Foot $\frac{1}{2}$ by 16 Foot $\frac{1}{2}$.

Now when you come to Measure the Walls of a Building, some are thicker, others thinner than one Brick and $\frac{1}{2}$, which several thicknesses must all be brought to the common thickness of one Brick and half, thus.

Take the Dimensions as they come in order, whether they be thicker or thinner, and set them down according to the following Method, with the thickness over each Dimension, then cast up the Content of each Dimension in square Feet and Inches, after which reduce it to half a Brick thick; thus if the Dimension is half a Brick thick, set it down as the Product gives it; if it be 1 Brick thick, multiply the Product of your Dimensions by 2; If the Product be one Brick and half multiply it by 3; if it be 2 Bricks thick multiply the Product by 4, *&c.*

This done, summ up all the Dimensions, it give the total Summ in square Feet of half a Brick thick, which total divide by 3 will give the Content in Feet of 1 Brick and $\frac{1}{2}$ work; then if you divide the Content by 272, omitting the 36 odd Inches, the Quote will give you the Content in square Rod of Brick and half Work.

½ B.	F. I. P.		
8. 2		3) 1737 (579 *Foot of*	
6. 5	52.4.10	B ½ *work.*	
2 B.	272) 589 (2 *Rod* 45 *Foot. Cont.*		
11. 4		45	*of the ſaid Dim.*
2.11	132.2.8.		
3 B ½		*The odd* 11 *Inches and* 2	
17. 9		*Parts we reject when we*	
8. 4	1035.5.0	*Divide by* 3.	
1 B.			
26. 4			
9.10	517.10.8		
	1707.11.2		

Some Meaſurers caſt up their Dimenſions thus, they multiply their Dimenſions together, and ſet the Content of each Dimenſion againſt it in its proper thickneſs; *that is* without reducing it to ½ Brick work: Then they take out all the Contents of the ſame thickneſs and add them together; After which reduce them to ½ Bricks, and then to Brick and ½, which done, add them and the Brick and ½ work all together; the Summ divide by 272, the Quote is Rods of Brick work in the whole at Brick and half thick

thick; which of these two ways you like best, that use in Practice.

Notes.

First, In Brick woork we seldom take the Dimensions nearer than Inches; *that is*, we do not measure to Halves or Quarters, unless in very fine Work.

Secondly, 'Tis evident that 272 Foot $\frac{1}{4}$ is a square Rod; but in dividing for Rod, we omit the Quarter, and divide by 272 only, the Quarter being neglected to make the Division more easie, and also for the parts of Inches not set down when you look the Dimensions.

Thirdly, Sometimes Walls have Water Tables, which are two Inches thicker than the forementioned thicknesses; In such case measure the Wall at the same thickness 'tis above the Water Table; and instead of setting down the Dimensions of two Inch work, set down but half one of the Dimensions.

For Example, Suppose the Dimensions of a Water Table or piece of two Inch Work be $\begin{matrix} 18.6 \\ 12.2 \end{matrix}$ then halve one of the Dimensions, and it will be a Dimension of 4 Inch Work,

that

that is, of Work half a Brick thick, and will ſtand thus, $\frac{1}{2}$ B.

18. 6

6. 1

Fourthly, In Meaſuring of Chimneys do thus; If the Chimney ſtand alone, *that is*, not leaning againſt, or being in a Wall, girt it about below the Mantle, or meaſure the Back and one Jaum and double it, and take that for the length, and the height of the Room for the breadth; theſe two Dimenſions multiplied together, gives you the Content in the ſame thickneſs the Jaums are.

But if the Chimney ſtand againſt the Wall of the Houſe which was meaſured before, then you muſt girt it round to the Wall, and multiply that into the height of the Room, the Product gives the Content. And here take notice that no deduction is made for the Vacancy betwixt the Hearth and the Mantle-tree, becauſe of the wieths and thickning of the next Hearth above.

Fifthly, For the Chimney Shafts, you muſt girt them round in the ſmalleſt part for the breadth, and take their height for the length; theſe two multiplied together gives a Content in the ſame thickneſs as their breadth; for they are always meaſured as if they were one ſolid piece of Brick work, in conſideration of the Pargeting and

and Scaffolding, which is required to bring them up.

Sixthly, In the measuring of Brick Arches we take the breadth of the Arch, and half the breadth and add them together, for the true breadth or measure of the Curve of the Arch, this multiply'd by the length gives the Content in the thickness of the Arch; If there be several Arches together, some Allowance must be made to the Workman, for the Vacuities or Spandrels, which are commonly fill'd up.

Seventhly, Note that all kind of Ornamental Work, as Architrave, Frieze, Cornice, Arches over Doors and Windows, Rubb'd Returns, Rustick Quoines and Mouldings, are generally measur'd by the Foot running Measure.

Eighthly, Peers, Necks, Pilasters, and such like work, is valued at so much a piece.

Ninthly, *Tyleing* is measur'd at so much the square, *that is*, 10 Foot every way, or 100 square Feet; only *Hyps*, *Valleys* and *Water-Courses*, are measured by the Foot running Measure.

Lastly, Paving with Brick is commonly done by the Square Yard.

In-

Inſtructions for Meaſuring of Carpenters Work.

The greateſt part of Carpenters work, as Flooring, Rooſing, Partitioning and Ciel Joyſts are meaſured by the Square, which Square contains 100 ſq. Feet; Or it is ſuch a piece of Work as is juſt 10 Foot long and 10 Foot broad; and in Bills of Meaſurement is generally Rated at ſo much the Square.

Windows are uſually ſet down at ſo much a Light, or ſo much a piece.

Door Caſes at ſo much a piece, either within or without Doors.

Mantletre and *Tarſels* at ſo much a piece.

Rayling, *Paleing*, *Lintoling*, *Guttering*, *Cornice* and *Window Board*, at ſo much the Foot running Meaſure.

Stairs at ſo much the Step, or ſo much a piece.

Inſtructions for Meaſuring of Glaziers Work.

The readieſt way of Meaſuring Glaziers work is with a ſliding Rule, by which the

Dimensions are taken very exact, even to $\frac{1}{4}$ of an Inch; they commonly agree for their Work by the Foot, whether the Glass be old or new, Squares or Quarries.

And here note, That if in measuring a Window, there be more Lights than one of the same bigness; as suppose you were to measure an eight Light Window, and each of the upper Pains were 3 Foot 2 Inches high, and 2 Foot 1 Inch broad, and the lower Pains were 8 Foot 6 Inches high, and 4 Foot 8 Inches broad, then you need make but two Dimensions of these 8 Pains of Glass, by setting them down as followeth.

F.	I.	
4 :	2	4 times.
2 :	1	
8 :	6	4 times.
4 :	8	

Sash Windows are Glazed by the Square; *that is*, we tell how many Squares there is in all the Lights, and then reckon what they come to, either at so much the Score, or so much the Dozen.

Instructions for Measuring Joynery, Painting Plaistering and Paving Work.

Joyners, Painters, Plaisterers and Paviers Works are generally agreed for by the Yard; and therefore having cast up your Dimensions, bring their whole Summ into Yards, by dividing the Feet by 9.

Now in taking the Dimensions of Joyners or Painters work, such as *Pollection* and *Bead* work, you must by a Line girt the Bends and Hollows, and so bring it down to the bottom; after which measure the length of your Line by your Rule, setting down this length for one of your Dimensions, then measure the length of the former height, and set down that for a second Dimension.

Note that Window-Shuts and Doors are measured work and half, because they are work'd on both sides.

The Painting of Windows are generally agreed for at so much the Light, and Casements at so much a piece.

Plaistering is also measur'd by the square Yard, and when a Sommer or Girdet lies below the Cieling it is commonly deducted, where the Workman finds Materials.

The Measuring of Masons Work.

Masons Work, whether it be superficial or solid, is measured by the Foot; if you measure the Workmanship only of Stone Steps, then you must measure both ends of the Step, as well as its length.

Of Deductions in general.

First, In Brick work, you are to deduct for all Doors, Windows, and other Vacancies that are measured in.

In Carpenters work, particularly Flooring, you must make deductions for the Well-hole for the Stairs, also for the Chimney Hearths, and the like.

Also in Partitioning, deduction must be made for all Doors and Windows that are measured in.

In Roofing we make no deductions for Window Shafts or Sky Lights, because they are more trouble to the Workman, than the Stuff is worth that would cover them.

In Plaisterers work, where the Workman finds Materials, there deduction is to be made for all Vacancies which are not Plaistered and yet are measured in.

In

Instructions for Measuring, Digging of Cellars, Vaults, Draines, *or the like.*

Digging is generally agreed for by so much the Yard solid Measure; and therefore, for finding the quantity of this kind of work three Dimensions must be taken, which are set down, and cast up as followeth.

F.	I.
26.	8
	8. 6
	4. 9
26.	8
	8. 6
208.	
	5. 4
	13. 4
226.	8
	4. 9
904.	
	2. 8
169.	5
1076.	7 *Solid Content in* F.

Y. F. I.

27) 1076 (39. 23. 7. *Solid Content.*

266

23

and after this method are all ſolid Dimenſions caſt up.

And here Note, that a Solid Yard of Earth, is accounted one Load.

Having thus given you a particular account of meaſuring moſt ſorts of Works, I ſhall in the next place give a ſhort Specimen of drawing out a Bill of Meaſurement.

Brick-

Bricklayers Work done for A. B. *at his Houſe in* C. D. *by* E. F.

	l.	*s.*	*d.*
For 50 Rod 68 Foot of Brick work, reduc'd to Brick and half, at 5 *l.* the Rod.	251	05	00
For 17 Square 75 Foot of plain Tyleing at the Square, at 2 *l.* the Square.	35	10	00
For 250 Foot of Rub'd Return, at 3 *s.* the Foot.	37	10	00
For 75 Foot of ſtrait Arch at 2 *s.* the Foot.	7	10	00
For 36 Foot of Jack Arch, at 2 *s.* 3 *d.* the Foot.	4	01	00
For 1679 Foot of Paveing with 10 Inch Tyle, at 7 *d.* the Foot.	48	19	05
The Summ.	384	15	05

Meaſured, *July* 27, 1699.
By me H. I.

After this manner, changing what is to be changed, are Bills of Meaſurement for all other ſort of Works formed and worded.

Directions for Measuring Board and Timber.

Board and Plank is commonly Measured by the Foot square; and therefore for finding the Content of any piece of Board, only two Dimensions are requisite to be taken, which are set down and cast up as in the preceding Examples has been taught.

Dimensions.

24. 8 } 47. 3. 4.
1. 11 }

13. 10 } 12. 8. 2.
0. 11 }

If a piece of Board or Plank be broader at one end than the other, add both ends together, and take half that Summ, or else Measure it in the middle for the true breadth, and Multiply it by the length for the Content.

But suppose a piece of Board or Plank be 3 Inches broad, and you would know how much in length will make a Foot square.

Then

Then becauſe a ſquare Foot contains 144 ſquare Inches; therefore if I divide 144 by 3, the Quote 48 is the Inches in length, which of that breadth will make a Foot ſquare.

Hence if you divide 144 by any given breadth, you get the length that will make one Foot ſquare to that breadth, the reaſon of which is evident; for if you Multiply the Diviſor and Quote, that is the length and breadth, the Product will make 144, the ſquare Inches in a ſquare Foot.

And after this manner is the Table upon the common two Foot Rules made, call'd the Table of Board Meaſure, which ſhows upon any breadth how much in length will make a Foot ſquare.

A Table of board Meaſure.

1	2	3	4	5	6	7	8	*Inch.*
12	6	4	3	2	2	1	1	*Foot.*
0	0	0	0	4.8	0	8 4/7	6	*Inch.*

By this Table it appears that if a piece of Board be 1 Inch broad, 'twill require 12 Foot in length to make a Foot ſquare; If 2 Inches broad, 6 Foot in length will make a Foot ſquare; If 8 Inches broad, 1 Foot 6 Inches will make a Foot ſquare.

To

To Measure Board by Gunter's Line.

Exam. 1. Suppose a piece of Plank be 196 Inches long and 22 Inches broad, What's the Content.

Extend the Compasses from 1 to 196 the length, the same Extent laid the same way, will reach from 22 the breadth to 4312, the Content in superficial Inches:

But if you will have the Content in Feet, then extend the Compasses from 144 to 196, the same Extent laid the same way, will reach from 22 to 33 Foot near, which is the Content of the Plank.

If the Dimensions be given in Feet and Inches, make an allowance for the odd Inches in parts of a Foot, by taking so many Feet and a half, or so many Feet and a quarter, *&c.*

Exam. 2. Suppose a piece of Board be 16 Foot 9 Inches long, and 2 Foot 3 Inches broad, How many square Feet is contained therein.

Extend the Compasses from 1 to 2 Foot $\frac{1}{2}$ the breadth, the same Extent will reach from 16 Foot $\frac{1}{2}$ to 37 Foot $\frac{3}{4}$ the Content near.

Exam. 3. A piece of Board is 17 Inches broad, and 34 Foot long, How many square Feet is contained therein. Be-

Becauſe the Breadth is given in Inches, and the Length in Feet, extend the Compaſſes from 12 to 17 the Breadth, the ſame Extent laid the ſame way, will reach from 34 Foot the length, to 48 Foot $\frac{1}{8}$ the ſuperficial Content.

Exam. 4. Suppoſe a piece of Board be 10 Inches broad, how much in length will make a Foot ſquare.

In this caſe you muſt extend your Compaſſes from 10 the breadth to 12, the ſame Extent being laid the ſame way, will reach from 12 to 14 Inches $\frac{2}{5}$, the length that will make one Foot ſquare.

The Meaſuring of Timber.

Timber is meaſured by the Foot ſolid Meaſure; and therefore for Meaſuring of ſquar'd Timber three Dimenſions muſt be taken, *that is*, Length, Breadth and Depth, by which Dimenſions the Content in ſolid Feet of any piece of Timber may eaſily be found.

And here note, That whether Timber be round or ſquare, if it be but as big at one end as it is at the other, it may eaſily and exactly be meaſured; but if it be bigger at one end than the other, then whether it be round or ſquare, there is trouble in Meaſuring it exactly by Arithmetick.

Now

Now before I proceed any farther, it will be neceſſary to ſhew you ſome Errors that are frequently committed in Meaſuring Timber.

Errors in Timber Meaſure.

Firſt, When a piece of Timber is not truly ſquare, but broader than 'tis thick, 'tis common with many Artificers in ſuch a caſe to add the breadth and thickneſs together, and take half the Summ for the true ſide of the Square, which is apparently falſe.

For Suppoſe a piece of Timber be 13 Inches broad and 3 Inches thick, then ſay they, the true ſide is 8 Inches, for 'tis half of 13 and 3 added together, which is manifeſtly falſe, for 8 times 8 is 64, and 3 times 13 is but 39; ſo that by this method the Area of the baſe is 25 Inches more than the truth, which Error multiplied by the length will Encreaſe it conſiderably; ſo that Timber Meaſur'd this way makes it more than really it is.

There is alſo a ſecond Error, which is more frequent than the former, committed in the Meaſuring round Timber, and that is this.

They

They take $\frac{1}{4}$ of the Girt of a piece of round Timber for the ſide of a Square equal to ſuch a Girt. which that 'tis falſe is very apparent.

Let the Girt of a piece of Timber be 70 Inches, What's the ſide of a Square equal thereto: Anſwer this way 17 Inches $\frac{1}{2}$, But in truth is very near 20 Inches; for the ſquare Root of the Area of a Circle, whoſe Circumference is 70 Inches. is 19 Inches $\frac{9}{10}$. Now 20 times 20 is 400, and $17\frac{1}{2}$ by $17\frac{1}{2}$ gives $306\frac{1}{4}$, which multiplied by the Length, will give a Content much leſs than the truth; ſo that Timber meaſur'd this way, makes it much leſs than it ſhould be.

There is yet a third Error, in meaſuring both round and ſquare Timber, that is bigger at one end than the other, and that is girting it in the Middle, which Cuſtom is alſo very Erroneous, as we ſhall examin in the following Example.

Suppoſe a piece of Timber be 26 Inches ſquare at the greater end, and 18 Inches ſquare at the leſſer end, and 36 Foot long, What's the ſolid Content.

By the Common or Falſe way.

26 *ſide great end.*
18 *ſide little end.*

44 *Summ of the ſides.*

22 *half ſum or ſide meaſur'd in the mid.*
22

44
44

484 A. *of the mean Square.*
432 *length in* I.

968
1452
1936

209088 *Solid. in Inches.*

By the True way.

26		18	
26		18	
156		144	
52		18	
676	A. *great base.*	324	A. *little base.*
324	A. *little base.*		
2704			
1352			
2028			

Root.

)219024 (468

676 A. *great Base.*

86) 590 324 A. *little Base.*

928) 7424 1468

144 *a third of the length.*

5872

5872

1468

211392 *True solid Content in* l.

209088 *False solid Content in* l.

02304 *Difference.*

Hence

Hence you ſee the common method gives a Content too little, and therefore if it happens to unite with the former of the two preceding Examples, then it is not ſo great, for that being too much and this too little, do in ſome meaſure mitigate the Error.

But in caſe the Piece be round, and you girt it in the middle; a double Error meets and makes the Content conſiderably leſs than it ſhould be; for the ſame Error happens in girting a round Tree in the middle, as in meaſuring the ſide of a ſquare Tree in the middle.

Having thus given you an account of theſe Errors, I ſhall in the next place propoſe ſome Examples, and then give you a Method that will in great meaſure remedy the two laſt of the preceding Errors, and therefore ſerve very well for practice.

Exam. 1. Suppoſe a piece of Timber is 2 Foot 3 Inches ſquare, and 20 Foot long, What's its Content.

2.3 7
2.3 5 *ſide.*

4:6
6:9

5:0:9
20 *length.*

101:3:0 *the ſolid Content.*

Exam.

Exam. 2. Suppoſe a piece of Timber be 1 Foot 2 Inches thick, 3 Foot 4 Inches broad, and 12 Foot 6 Inches long, What's the Content.

1 : 2 *thickneſs.*
3 : 4 *breadth.*

3 : 6
4 : 8

3 : 10 : 8
12 : 6 : 0 *length.*

46 : 8
1 : 11 : 4

47 : 7 : 4 *Solid Content.*

Next for Tapering Timber, whether it be ſquare or round, inſtead of taking its Dimenſion in the Middle, Meaſure or Girt it at ſomething more than $\frac{1}{3}$ of the length from the greater end, and inſtead of taking $\frac{1}{4}$ of the Girt for the ſide of the ſquare equal, take $\frac{2}{7}$ of it; Theſe ways though not exactly true, yet will in great Meaſure correct the former, and ſerve very well for ordinary Buſineſs, or where great exactneſs is not required.

Exam. 3. Suppose a piece of Timber, at $\frac{1}{3}$ from the greater end Measures 1 Foot 5 Inches square, and suppose it be 23 Foot long, What's the Content

F. I.
1. 5 } *side.*
1. 5 }

1. 5
5
2. 1

2. 0. 1
23 *length.*

46. 1. 11 *Solid Content.*

Exam. 4. Suppose the Girt of a Tapering Tree, at $\frac{1}{3}$ from the greater end, be 63 Inches, and its length 25 Foot, What's the Content.

Here instead of taking one Quarter of 63, which is 15 and $\frac{3}{4}$ I take $\frac{2}{7}$, which is 18 Inches, or 1 Foot 6 Inches.

Feet

F. I.
1. 6
1. 6

1. 6
6
3

2. 3
25. *length.*

50
6. 3

56. 3 *Solid Content.*

Exam. 5. A piece of Timber is 16 Inches one way, and 4 Inches the other, How much in length will make a Foot solid.

16
4

64 *Inches.*

64) 1728 (27 I.
448

So that 2 Foot 3 Inches in length will make a Foot solid.

From this Example 'tis evident, when a piece of Timber is either ſquare or thicker one way than the other, How much in length will make a Foot ſolid: For if you multiply the ſide of the ſquare by it ſelf, or the thickneſs and breadth together, and by this Product Divide 1728 the ſolid Inches in one ſolid Foot, the Quote will give you the length that will make one ſolid Foot.

The reaſon of which is very plain; for the breadth multiplied into the thickneſs is the Diviſor, and that multiplied by the Quote, which is the length, will make 1728, the Inches in one ſolid Foot.

And thus is the Table upon the common two Foot Rules made, call'd the Table of Timber Meaſure, which ſhows how much in length of any exact Square piece will make a Foot ſolid.

A Table of Timber Meaſure.

In. Sq.	1	2	3	4	5	6	7	8	9	10
Feet.	144	36	16	9	5	4	2	2	1	1
In. P.	0	0	0	0	$9\frac{3}{25}$	0	$11\frac{23}{49}$	3	9 $\frac{1}{3}$	$5\frac{7}{25}$

The

The uſe of this Table is thus, If a piece of Timber be 3 Inches ſquare, it ſhews you that 16 Foot in length will make a Foot ſolid; If it be 7 Inches, then you will find that 2 Foot 11 Inches and $\frac{1}{4}\frac{3}{9}$ of an Inch in length, will make a Foot ſolid.

To Meaſure Timber by Gunter's *Line.*

Exam. 1. Suppoſe a piece of Timber is 10 Inches ſquare, and 24 Foot long, What's the Content in Feet.

The Extent upon the Line of Numbers from 12 to 10 the Inches ſquare, being twice repeated the ſame way from 24 Foot the length, gives 16 Foot $\frac{3}{4}$ *prope* the Content.

Exam. 2. A piece of Timber is 8 Inches one way, 13 Inches the other, and 19 Foot long, What's the Content.

Reduce the piece to a ſquare thus, Take the Middle betwixt 8 and 13 it gives 10 Inches $\frac{1}{4}$ near, for the ſide of the Square.

Then the Extent from 12 Inches to 10 Inches $\frac{1}{4}$, being twice repeated the ſame way from 19 the length, will reach to 13 Foot $\frac{1}{4}$ near, the Content required.

Or thus, without bringing it to a ſquare. The Extent from 12 to 8 Inches the Depth, laid the ſame way from 13 the breadth, will reach to 8 $\frac{3}{4}$ a fourth Number: Then the Extent from 8 $\frac{3}{4}$ to 12, ſhall reach the ſame way from 19 the length to 13 Foot $\frac{3}{4}$, the ſame Content as before.

Exam. 4. Suppoſe the Girt of a piece of Timber be 56 Inches and 27 Foot long, What's the Content.

Having taken $\frac{2}{7}$ of 56 the Girt, which is 16 Inches for the ſide of the ſquare equal, proceed as in the firſt Example; *that is*, extend the Compaſſes from 12 to 16, this being twice repeated from 27 the length, will reach to 48 Foot the Content.

Exam. 4. Suppoſe the ſide of a ſquare piece of Timber be 6 Inches ſquare, How much in length will make a Foot ſolid.

The Extent from the ſide of the ſquare to 12 Inches, laid twice the ſame way from 1, ſhall reach to the Feet and parts of a Foot, that in length will make a Foot ſolid, in our Example 'twill reach to juſt 4 Foot.

If the ſide of the ſquare be more than 12 Inches, ſuppoſe 14; then the Extent of the ſide of the ſquare to 12, being twice repeated the ſame way from 12, ſhall reach to the Inches and parts of an Inch, that in length will make a Foot ſolid; in this Example 'twill reach a little beyond 10; ſo that very near 10 Inches in length will make a Foot ſolid.

If

If your Timber be Tapering, then whether it be ſquare or round, take the Dimenſions of it at $\frac{1}{3}$ from the greater end, as before has been directed.

Of the Tranſmutation of Figures.

To reduce any ſuperficial Figure to a Square.

Rule. *Extract the ſquare Root of the Area of any Superficial Figure, it gives the ſide of a Square equal thereto,*

Exam. 1. Suppoſe the Area of a Circle be 2346, What's the ſide of a ſquare equal thereto. *Anſw.* 48 near.

Exam. 2. Suppoſe the Area of a Square be given, and 'tis required to find the ſide of another ſquare 8 times as big.

Rule. *Multiply the given Area by the number of times you would have it bigger, the ſquare Root of the Product is the ſide of the Square required:* If you would have it but half or $\frac{1}{4}$, or any other part of a Tine ſo big, then Extract the ſquare Root of a Half or Quarter, *&c.* of the propoſed Area, it gives the ſide of a Square required.

To reduce any Superficial Figure to a Rectangle or long Square, that ſhall have any aſſigned length or Breadth.

Rule. *Divide the given Area by the propoſed length or breadth, the Quote is the length or breadth required.*.

Exam. 3. Suppoſe the Area of a Triangle be 366 Foot, What's the breadth of a Rectangle equal thereto, whoſe length is 61 Foot: *Anſw.* 6 Foot,

To reduce any ſuperficial Figure to a Triangle, whoſe Baſe or Perpendicular ſhall be any aſſigned Dimenſion.

Rule. *Divide the propoſed Area by half the given Quantity, the Quote is the Perpendicular or Baſe required.*

Exam. 4. Suppoſe the Area of any Figure be 864 Foot, what is the Baſe of a Triangle equal thereto, whoſe Perpendicular is 36 Foot. *Anſw.* 48.

To reduce any Superficial Figure to a Circle, *that is*, to find the Diameter of a Circle, equal to any given Superficies.

This is no more than what was ſhown in page 184; *that is, having the Area of a Circle to find its Diameter.*

And

And therefore if you multiply any given Area by 1.27324, and extract the ſquare Root of the Product, it will be the Diameter of the Circle required.

To reduce any Superficial Figure to an Elipſis, that ſhall have any aſſigned breadth or length.

Rule. *What Figure ſoever the given Area is, take it for the Area of a Circle, to which find the ſquare of the Diameter, by the method ſhown in* page 184. *then divide the ſaid ſquare by the given length or breadth of the Elipſis, the Quote will be the breadth or length of the Elipſis required.*

Suppoſe the Area of a Circle be 6432, What's the length of an Elipſis equal thereto, whoſe Conjugate, Diameter or Breadth is 46 Foot. *Anſw.* 89 Foot *near.*

To find the ſide of a Cube, equal to the ſolid Content of any Body.

Rule. *Extract the Cube Root of the given Solid Body, and it gives the ſide of a Cube equal thereto.*

Suppoſe the ſolidity of a Parallelepipedon be 32768 Foot, What's the ſide of a Cube equal thereto. *Anſw.* 32.

If you would find the ſide of a Cube double, treble, *&c.* or a half, a third, *&c.*

'tis

'tis but doubling, trebling, &c. or halving thirding, &c. the propoſed Area, and then Extracting the ſquare Root of ſuch Product or Quote, and it gives the ſide of the Cube required.

To find the Diameter of a Sphere equal to any given Solidity.

This is no more than what was effected in *page* 229, where the ſolidity of a Sphere was given to find its Diameter; the difference is only this, whatever the given Solidity is, take it for the ſolidity of a Sphere, and find a Diameter to it, as is taught in the foremention'd Page.

Laſtly, 'If you would reduce any Superficial figure to any other ſuperficial right lin'd 'Figure, 'tis done both for Regular and Irregular Figures univerſally by *Theo* 28. to 'the uſe of which I refer you for an Example.

'The like alſo, for Encreaſing or Augmenting of Solids in a given Ratio, is ſhewn in 'the Uſe *Theo*. 45. to which I alſo refer you.

How to make a Veſſel of any Form, to Hold any Quantity of Liquor, Corn, &c.

Before I paſs to this neceſſary part of Geometry, it will be requiſite to inform you that,

282 Cubick Inches is 1 Ale or Beer Gall.
272 Cubick Inches is 1 Corn or Dry Gal.
231 Cubick Inches is 1 Wine Gallon.

And therefore,

Cub. In.	makes		
2538		1 Firkin	Beer.
5076		1 Kilderkin	
10152		1 Barrel	
2256		1 Firkin	Ale.
4512		1 Kilderkin	
9024		1 Barrel	
4158		1 Rundlet	Wine.
14553		1 Hogshead	
19404		1 Tercion	
29106		1 Pipe	
58212		1 Tunn	

Dry Measure.

Cub. In.	make	
544		1 Peck.
2176		1 Buch. Land Measure.
2720		1 Buch. Water Measure.
54400		1 Hundred of Lime, or 25 B.
78336		1 Chaldron, or 36 B.

To find the side of a Cube, to hold any Quantity of Liquid or Dry Measure.

Rule. *Find the Cube Inches in the Vessel proposed, the Cube Root of which is the side of a Cube equal.*

Exam. 1. What's the side of a Cube that will hold Eight Beer Barrels.

10152 Inches in 1 Beer Barrel.
8

81216 Cube Inches in 8 Beer Barrels.

the Cube Root of which is 43.4 near: the side of a Cube that will hold Eight Barrels of Beer.

Exam. 2. What's the side of a Cube that will hold one Chaldron of Coals.

2720 Inches in a Buch. Water Measure.
36

16320
8160

97920 Cubick Inches in a Chaldron.

the Cube Root of which is 46.2 near, the side of the Cube that will hold 1 Chaldron.

If you would make a Parallelepipedon or Prism of any Limited Base, whether square

or

or oblong, or of any assigned height to hold any Quantity; *Then divide the whole Solidity by the Area of the given Base, or by the height, the Quote is the height or Area of the Base, the square Root of which is the side required.*

Exam. 3. What's the side of a square Base of a Parallelepipedon, whose height is 6 Foot 9 Inches, that will hold 6 Chaldron of Coals.

78336 Cubick Inches in a Chaldron.
6

470016 Cubick Inches in 6 Chaldron.

F. I.
6, 9
12

81 Inches heigh.

81) 470016 (5802 Area of the Base.
650
216
54

Note.

Note. That for the 54 Remaining, I add 4 Tenths to the Quote, ſo that the Area of the Baſe is 5802.4 Inches, the ſquare Root of which is 76.1 Inches *near* the ſide of the Baſe.

Exam. 4. I would have a Ceſtern, the Length of which let be 74 Inches, the Breadth 26 Inches, How High muſt it be to hold 6 Barrels of Water.

Rule. *Find the Cubick Inches in 6 Barrels, then by the Product of 74 Inches into 26, that is, by 1924. Divide the aforeſaid Cubick Inches, the Quote is the Height of the ſaid Ciſtern.*

Work thus, and you ſhall find that 31.7 *near* will be the height of ſuch a Ciſtern.

Exam. 4. Of what Diameter muſt a Cylendrick Veſſel be, whoſe height is 3 Foot, to hold two Barrels of Water?

Rule. *Find the Cubick Inches in two Barrels, Divide thoſe Inches by the height of the Cylender, the Quote is the Area of the Baſe; which being a Circle, find the Diameter as is directed in* page 184.

Thus proceed, and you ſhall find the Diameter of the Cylender to be 26.4 Inches *near.*

From theſe few Examples and what preceeds, 'tis evident how to change any kind of Veſſel into any other kind; as alſo to make a Veſſel of any ſhape, to hold any Quantity of Liquor, Corn, *&c.*

I ſhall here Incert two Tables of the Proportion betwixt the Weights and Magnitudes of Metals; the firſt ſhewing the Proportion of the Weight of each Metal to one another, when the Bodies or Figures are equal: The ſecond ſhewing the Proportion of the ſides of like Solids, when the weight of the Metals are equal.

Firſt Table.

Tin	1554
Iron	1680
Copper	1890
Silver	2030
Lead	2415
Quick-ſilver	2850
Gold	3990

The firſt Table ſhews you that if any Body of Tin, ſuppoſe a Cube weigh 1554 Grains, the like Body of Gold of the ſame Bigneſs ſhall weigh 3990 Grains.

The Second Table.

Gold	3895
Quick-ſilver	5433
Lead	6435
Silver	7161
Copper	8222
Iron	9250
Tinn	10000

This

This Second Table is to ſhew you the Proportion of the Sides when the Weights are equal. As ſuppoſe a piece of Gold and a piece of Tin of equal weight, then if two Cubes be made of theſe two pieces of Metal, the ſide of the Cube of Tin, ſhall be to the ſide of the Cube of Gold, as 3895 to 10000.

How

How to take an exact Plot or Draught of a Court, Yard, Garden, *&c.*

THE Practice and Method of performing this, is grounded upon the Problem of making a Triangle with any 3 given Lines, provided any two of them taken together be longer than the third, See *Prob.* 1. with its use.

Supposing therefore the Artist be ready at constructing this Problem, I shall proceed to shew how to effect what I propose.

Let ABCDEFG be a Court or Yard, whose Plan is required; First take a view of it by casting your Eye quite round it, Then upon a piece of Paper delineate the shape of it as near as you can



T by

by the hand, It matters not how falſe or diſproportional the Lines are, provided you make the ſame number in the Draught as there is really in the Court or Yard, and that you put them nearly in the ſame ſcituation.

This done, with a Chain divided in half Feet, or elſe a 10 Foot Rod divided into Feet and Inches; upon a Line ſtrain'd meaſure the length of every ſide, *that is*, of the ſide A B, B C, C D, *&c.* ſetting down upon this rough Draught the Dimenſion or length of each ſide as you find it by Meaſuring.

For Example, the ſide A B is in length 56 F. 2 I. I therefore ſet down upon the line A B in the rough Draught 56 F. 2 I. In like manner the Line B C meaſures 72 F. 9 I. this I ſet down upon the Line B C, and ſo upon all the reſt.

Having thus rounded it, go to ſome corner, from whence you beſt may ſee, and moſt conveniently meaſure to all the other Angles; as in this caſe I go to the Angle C, and from thence with my Rod meaſure to the Angles A. G. F. E. which Lengths I ſet down upon the correſponding Lines in your rough Draught: The Dimenſions being thus taken, you may by help of the forementioned Problem, draw a fair and true Plan thereof, as follows.

How

How from this Rude Draught to draw a fair one.

Upon the Paper on which you deſign to draw it make a point as C, to repreſent the Corner from whence you meaſured the Diagonals; through this point draw a blind Line at pleaſure for your firſt Diagonal, on which lay off from C to A 111 Foot, 3 Inches, its meaſure taken from ſome Duodecimal Scale; for if you actually meaſure by Feet and common Inches, it muſt be taken from ſuch a Scale, then take the ſide A B 56 Foot 2 Inches from the ſaid Scale, and ſetting one Foot in A, with the other deſcribe an obſcure Arch toward C; next take the ſide C B 72 Foot 9 Inches from the ſaid Scale, and ſetting one Foot in C, with the other croſs the former Ark in B, then draw the Lines C B and A B in Black, Red or what Colour elſe you pleaſe, ſo have you gotten two of the Boundary Lines of your Draught.

Next, For your ſecond Diagonal C G being 128 Foot, take 128 Foot from your Scale, and ſetting one Foot in C, with the other make a blind Arch at G, then take 55 Foot the ſide A G, and ſetting one Foot in A, with the other croſs the former Arch in G,

and draw A G, ſo have you got a third boundary Line.

Again, take 92 Foot the third Diagonal, and ſetting one Foot in C, with the other ſtrike a blind Arch at F, and with the Diſtance G F 62 Foot,, and one Foot in G croſs the ſaid Arch in F and draw G F, ſo is a fourth Line of your Arch finiſhed.

This done, Take 120 Foot from your Scale, and with one Foot in C make a blind Arch at E; then take F E 42 Foot, and with one Point in F croſs the former Arch in E, and draw F E for the fifth boundary Line of your Draught.

Laſtly, take the ſide E D 83 Foot betwixt your Compaſſes, and with one Foot in E deſcribe an obſcure Arch at D; then take 79 Foot the length of the ſide C D, and with one Foot in C croſs the Arch in the point D; draw the Lines E D and C D, and your Draught is finiſhed.

This method may likewiſe be uſed for ſmall Fields or Encloſures.

Another way to do the ſame.

Sometimes it may happen, that by reaſon of old Stuff or Rubbiſh lying upon the Plot or

or Plan, Diagonals cannot be Measured; in such case the following Method will be very serviceable.

Let the Figure ABCD following be such a Ruinous Foundation, whose Plot is required.

Draw a rough Eye-Draught, and measure the several Boundary Lines, whose Measures set down upon the corresponding Sides as before; this done. go to the Angle A and measure upon the side A B 6 Foot to *g*, (or as occasion requires more or less) also measure upon the side A D 6 Foot to *e*, (or more or less) from *e* to *g* strain a Line and measure the length thereof, which let be 8 Foot; this set down upon the prick't Line *e g* in your Draught.

Do the like at the Angle B, and set these down also; so have you Dimensions sufficient to plot your Draught as followeth.

To draw a Fair Plot of the ſame.

Upon a Sheet of clean Paper draw a Line, and lay thereon from A to B 28 Foot, taken from your Scale.



Then take 6 Foot from the Scale, and lay from A to *g*, and from B to *h*; upon the Points A and B deſcribe two prick't Arches at *e* & *k*; then becauſe the Diagonal *g e* is 8 Foot, take 8 from the Scale, and with one point in *g* croſs the former Arch in *e*, from A through *e* draw a Line at pleaſure, and lay thereon from A to D 27 Foot.

Again, Becauſe the Diagonal *h k* is 9, I take 9 from my Scale, and letting one Foot in *h*, with the other croſs the former Arch in *k*; then from *B* through *k* I draw a Line at pleaſure, and lay thereon from B to C 26 Foot for the ſide B C: This done, from the points D and C draw the Line D C and the

the Plot is finiſhed, which if right, the Line D C will meaſure upon the Scale 21 Foot, as it really is.

Note, The prick't Lines *e g*, *b k* muſt not appear in a fair Draught.

Your Plot being fairly drawn, the Content is found in Feet and Inches by cutting it into Triangles, as is ſhewn in *Prop.* 4. of the preceding Treatiſe of Meaſuring; If the Content of it be required in *Perches*, *Roods* or *Acres*, then divide the Content in ſquare Feet, by 272.25 the Quote will be ſquare Perch; Or divide the ſquare Feet by 10890 the Quote will be ſquare Roods; Or if you divide by 43560, the Quote will be *Acres*.

THE MANNER OF CUTTING AND MEASURING THE FIVE *Regular Platonick Bodies.*

LET the Diam. of the Sphere be 2, then the ſides of the 5 Regular Bodies inſcrib'd in the ſame, will be as follows,

Diameter of the Sphere 2.
Side of the Tetraedron 1.6229.
Side of the Cube 1.1547.
Side of the Octaedron 1.4142.

Side

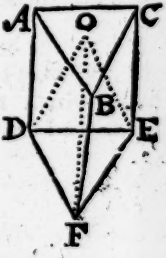
Side of the Dodecaedron 0.7136.
Side of the Icoſaedron 1.0514.

To cut the Tetraedron.

Make a Triangular Priſm, whoſe Baſe is an Equilateral Triangle, and ſide of which muſt be 1.62, *&c.* and the height of the ſaid Priſm $\frac{2}{3}$ of the Diameter of the Sphere, *that is*, $\frac{2}{3}$ of $2 = \frac{4}{3}$ or $1\frac{1}{3}$. This done, find the Center of the Triangle (*Prob.* 17.) then let Plains be conceived to paſs through the Points DFO, EFO, DEO, *that is*, ſhave the Wood exact to theſe Points, and you'll form the Tetraedron required.

To Meaſure it.

This Body being a Triangular Pyramid, muſt be meaſured by the Rule for meaſuring a Pyramid, *viz.* by multiplying any one of the Triangular Plains by $\frac{1}{3}$ of the neareſt diſtance betwixt one Plain and the point oppoſite, the Product is the ſolid Content.



Its ſuperficial Content is found by Multiplying the Area of any one of the Triangular Plains by their

their Number, which is 4, the Product is the Area of its Surface.

How to cut the Cube.

For the cutting of this Figure there needs no direction, only let the side be 1.1547 and plain it exactly square.

To Measure it.

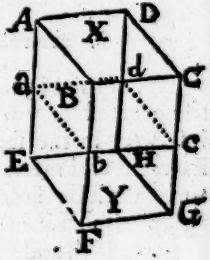
Multiply any one side by it self, and that Product again by it self, this last Product is the solid Content.

Its Superficial Content is found by Multiplying the Area of any one of the Bases by their Number, which is 6, the Product is the Superficial Content.

To cut the Octaedron.

Let there be made a Parallelepipedon ABCDEFGH whose Base is a square, the side of which must be 1.41, *&c.* and the height the Diameter of the Sphere, which is 2.

Bisect



Biſect the Lines AE, BF, CG, DH in the Points, *a*, *b*, *c*, *d*, and join thoſe Points by the prick't Lines *ab*, *bc*, *cd*, *da*; this done, find the Centers X. Y of both ſquare Baſes, then conceive Plains to paſs through the Points *ab*X, *bc*X, *cd*X, *da*X; *ab*Y, *bc*Y, *cd*Y, *da*Y; *that is*, plain it off to thoſe Points, and it will form the Octaedron required.

Note. I have not drawn the Lines of Section in this Figure, becauſe it would make too great confuſion, the thing being clear without it, for if your Plain paſſes but through the forementioned Points, you cannot help forming the Figure.

To Meaſure it.

This ſolid is only two Pyramids abuting upon one common Baſe, and is therefore meaſured by Multiplying one of its ſides by it ſelf, and that Product again by $\frac{1}{3}$ of the Diſtance of any two Corners, this laſt Product is the ſolid Content.

Its Superficial Content is found by Multiplying the Area of any one of the triangular Baſes by their Number, which is 8; the Product is the Content of the Surface.

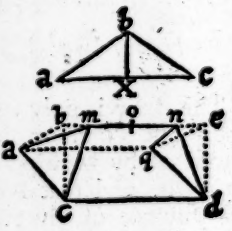
To

To cut the Dodecaedron.

To cut this Figure whose side is (0.71364) the greatest Segment of the side of a Cube inscrib'd in the said Sphere, when cut in Extream and mean Proportion. Make therefore a Cube fit to inscribe in the same Sphere, the side of which is 1.1547.

Let half the side of this Cube, *viz.* 5773 be cut in Extream and mean Proportion, then form an Isosceles, whose Base is *a c* the side of the Cube; and its Altitude *b x* (= .71364) the greater Segment of half the side of the Cube cut as aforesaid.

This done make a Triangular Prism *a b c q e d*; having this Triangle for its Base and the side of the Cube for its Altitude, bisect *b e* in *o*, and take *o m* and *o n* severally equal to *b x*, then let plains pass through the points *a c m* and *a d m*, *q d n*, *that is*, plain off the Wood to these points *a c m* and *a d n*, Make 6 of these Figures, which glew'd, or otherwise fasten'd to the Faces of the Cube, will make the Dodecaedron required.



To

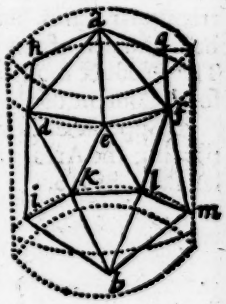
To Meaſure it.

This Body is made up of 12 equal Pyramids, the Baſes of which are the Pentagonal Plains that bound the Figure, and the tops of the Figure the Center of it; find therefore one of theſe Pentagonal Baſes, and multiply it by $\frac{1}{3}$ of its diſtance from the Center, this Product multiply by 12, it gives the ſolid Content of the Figure.

Its ſuperficial Content is found by multiplying the Area of one of the Pentagonal Plains by 12.

To cut the Icoſaedron.

Make a Cylender, the Radius of whoſe Baſe let be (0.8944) the ſquare Root of $\frac{4}{5}$ of the ſquare of the Diameter of the Sphere, and its Altitude (2) equal to the Diameter of the Sphere; find (*a* & *b*) the Centers of both the Circular Baſes; then from both theſe Circular Baſes on the curve Surface of the



Cylen-

Cylender, describe two Circles at $\frac{1}{5}$ of the Diameter distance therefrom. Divide any one of them, suppose the upper into 5 equal parts, and likewise the under Parallel after the same manner, only with this difference, that the beginning of the Divisions on this, may be perpendicular over the middle of the other Divisions; then let Plains pass through the Points *bda*, *dea*, *efa*, *fga*, &c. Also *dik*, *dek*, *kel*, *lef*, *lfm*, &c. *ikb*, *klb*, *lmb*, &c. and you will form the Icosaedron.

To Measure it.

This Figure is made up of 20 equal Pyramids, whose Bases are the Triangular Plains that bound the Figure, and height the half distance betwixt two opposite Faces; Find therefore the Area of one of these triangular Plains, and Multiply by $\frac{1}{3}$ of the half distance of any of the opposite Plains; The Product multiply by 20, it gives the solid Content of the Icosaedron.

Its Superficial Content is found, by Multiplying the Area of any one of the Trianlar Bases by 20.

The Proportions of the Sphere, and of the Five Regular Bodies inscrib'd in the same. As found in Peter Herigon. Cur. Mat.

Let the Diameter of the Sphere be 2.

The Circum. of the greatest Cir	6.28318
Superficies of the greatest Circle	3.14159
Superficies of the Sphere	12.56637
Solidity of the Sphere	4. 1879
The side of the Tetraedron	1.62299
Superficies of the Tetraedron	4. 6188
Solidity of the Tetraedron	0.15132
Side of the Cube	1. 1547
Superficies of the Cube	8.
Solidity of the Cube	1. 5396
Side of the Octaedron	1.41421
Superficies of the Octaedron	6. 9282
Solidity of the Octaedron	2.33333
Side of the Dodecaedron	0.71364
Superficies of the Dodecaedron	10.51462
Solidity of the Dodecaedron	2.78518
Side of the Icosaedron	1.05146
Superficies of the Icosaedron	9.57454
Solidity of the Icosaedron	2 53615

If you would cut one of these Bodies out of a Sphere of any other Diameter. Say, as the Diameter of the Sphere 2, is to the side of any one of the Solids inscrib'd in the same, suppose the Cube 1.1547; So is the Diameter of any other Sphere, suppose 8, to 9.2376, the side of the Cube inscrib'd in this latter Sphere.

FINIS.